Physics of the Atmosphere

Lecture 4: Static Forces in the Atmosphere (I)

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Forces (Basics)

- **What is the "Force"?**
  - Keywords:
    - Object
    - Push/Pull
    - Motion
    - Speed-up/slow-down
    - Direction (Change)
    - .
    - .
  - Definition
    - A quantity that acts on (push/pull) an object (mass) making the object move (speed-up/slow-down) to a specific direction.

Forces (Basics)

- **What is the "Force"?**
  - Math Expression:

$$\mathbf{F} = m\mathbf{a}$$

$\mathbf{F}$: force
m: mass
$\mathbf{a}$: acceleration

  - Physical Meaning:
    -

Forces (Basics)

- **What is the "Force"?**
  - Unit:

$$\mathbf{F} = m\mathbf{a}$$

$\mathbf{F}$: Newton = kg m s^{-2}
m: kg
$\mathbf{a}$: m s^{-2}

  - Dimension:

$$\mathbf{F} = m\mathbf{a}$$

$\mathbf{F}$: [N] = [MLT^{-2}]
m: [M]
$\mathbf{a}$: [LT^{-2}]

Forces (Basics)

- **Basic Physical Quantities (Dimensions & Units)**

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Quantity	Dimension	Units (SI)	Relationship
Length (l)	L	m	
Time (t)	T	s	
Mass (m)	M	kg	
Temperature	Θ	K	
Velocity (V)	LT^{-1}	m s^{-1}	$V = dl/dt$
Acceleration (a)	LT^{-2}	m s^{-2}	$a = dV/dt$
Force (F)	MLT^{-2}	kg m s^{-2}; N	$F = ma = dP/dt$
Momentum (P)	MLT^{-1}	kg m s^{-1}	$P = mV$
Work (W)	ML^2T^{-2}	kg m^2 s^{-2}; J	$W = F{\cdot}l$
Energy (E)	ML^2T^{-2}	kg m^2 s^{-2}; J	
Power (P)	ML^2T^{-3}	kg m^2 s^{-3}; J s^{-1}	$P = dW/dt$; dE/dt

u	$\frac{dx}{dt}$	m s^{-1}
v	$\frac{dy}{dt}$	m s^{-1}
$w(\omega)$	$\frac{dz}{dt}\left(\frac{dp}{dt}\right)$	m s^{-1} (Pa s^{-1})

Types of Forces

- **What kind of forces?**
  - Gravitational Force (Weight)
  - Electric Force
  - Magnetic Force
  - Centrifugal Force
  - Coriolis Force

  Apparent (Fictitious) Forces

  Body Forces

  - Pressure Force
  - Frictional (Viscous) Force

  Surface Forces

Types of Forces

- **Body Forces:**
  - Act at a Distance: Two objects are not in physical contact but exert a force (push/pull).
  - Long range forces capable of penetrating into the interior of the fluid and act on all elements.
  - No contact is necessary between the object and the force source.

Types of Forces

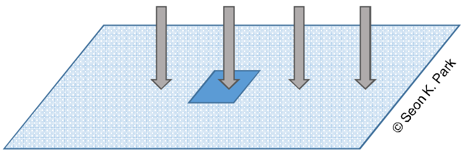
- **Body Forces:**
  - Gravitational Force (Weight)
    - A gravitational pull between the Earth and the Moon
    - You can jump up into the air but immediately fall down to the Earth.
  - Electric Force
    - An electrical pull between the protons in the nucleus and the electrons outside the nucleus of an atom
  - Magnetic Force
    - A magnetic pull between two magnets
  - Apparent forces due to coordinate system motion (e.g., rotating frame)

Types of Forces

- **Surface Forces:**
  - Forces acting when the two interacting objects are in physical contact
  - Short range forces that act only on a thin layer adjacent to the boundary of a fluid element.
  - Negligible when there is no direct mechanical contact between the interacting elements

Types of Forces

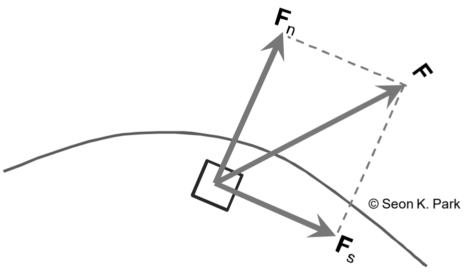
- **Surface Forces:**
  - Pressure (Normal) Force
    - Pressure is the normal force per unit area



$$p = \frac{F}{A}$$

Types of Forces

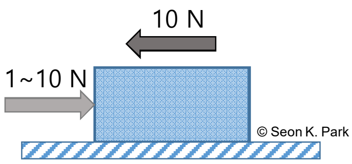
- **Surface Forces:**
  - Normal Force vs. Tangential Force

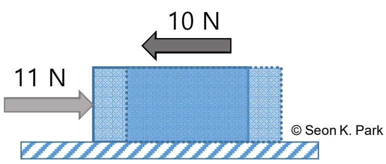


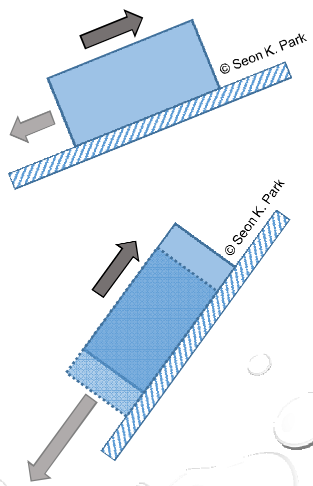
$$\mathbf{F} = \mathbf{F}_n + \mathbf{F}_s$$
$$\mathbf{F} = F_n\hat{\mathbf{n}} + F_s\hat{\mathbf{s}}$$

Types of Forces

- **Surface Forces:**
  - Frictional (Viscous) Force
    - Force exerted by a surface as an object moves across it
    - Sliding vs. Static Friction
    - Frictional force acts opposite to any applied force.

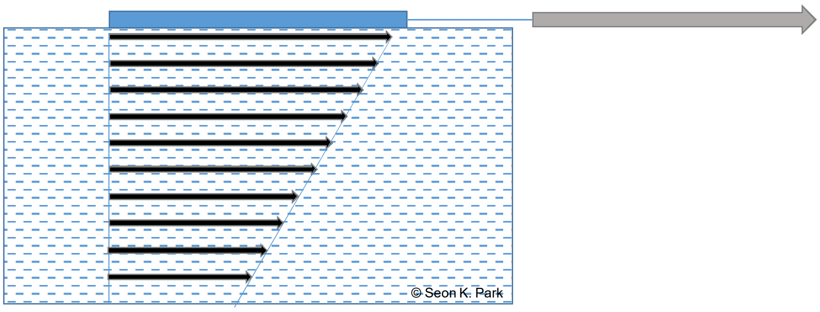






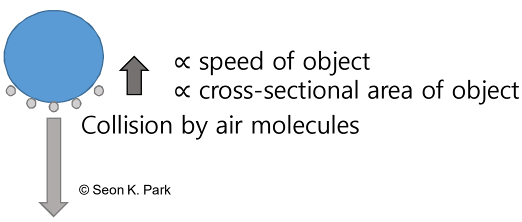
Types of Forces

- **Surface Forces:**
  - Frictional (Viscous) Force
    - In a fluid, frictional force depends on the viscosity



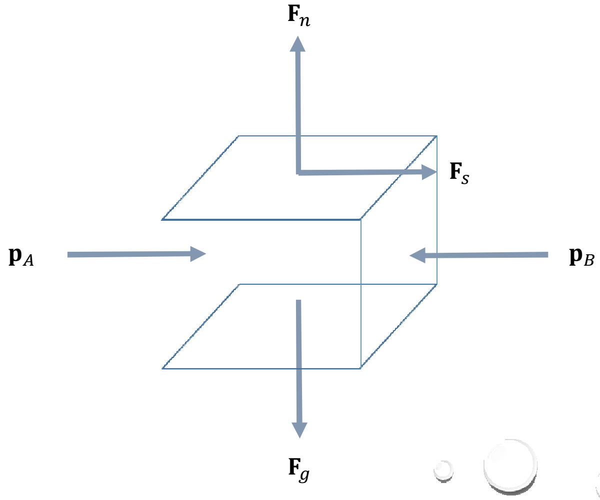
Types of Forces

- **Surface Forces:**
  - Air Resistance Force
    - Frictional force that acts upon objects as they travel through the air



Atmospheric Forces

- What kind of forces in the atmosphere?



Atmospheric Forces

- What kind of forces in the atmosphere?
  - Gravitational Force
  - Pressure Force
  - Pressure Gradient Force

  - Centrifugal Force
  - Coriolis Force

  - Frictional Force

Static vs. Dynamic Forces

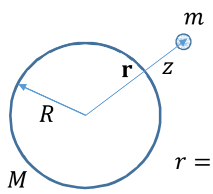
- **Static Forces:**
  - Present only in portions of atmosphere at rest relatively to the Earth
    - Gravitational Force
    - Pressure Gradient Force
    - Centrifugal Force
- **Dynamical Forces:**
  - Present only in portions of atmosphere moving relatively to the Earth
    - Coriolis Force
    - Frictional Force
- **Often, in atmospheric physics, forces are considered per unit of mass (i.e., accelerations)**

$$\mathbf{F} = m\,\mathbf{a} \longrightarrow \frac{\mathbf{F}}{m} = \mathbf{a}$$

Gravitational Force

- **Definition:**
  - The sole body force on atmospheric air parcels is due to gravity.
  - Newton's law of universal gravitation: any two elements of mass in the universe attract each other with a force proportional to their masses and inversely proportional to the square of the distance separating them.
- **Formulation:**

$$\mathbf{F}_g = -\frac{GMm}{r^2}\left(\frac{\mathbf{r}}{r}\right)$$



$r = |\mathbf{r}| = R + z \approx R$

Gravitational Force

- **Formulation:**

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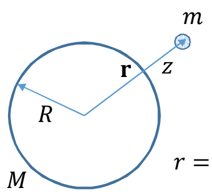


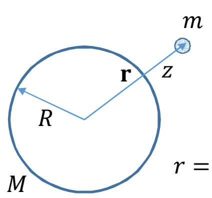
$$r = |\mathbf{r}| = R + z \approx R$$

- G = universal gravitational constant = 6.67×10^{-11} m^3 kg^{-1} s^{-2}
- $M \sim 5.97 \times 10^{24}$ kg (mass of the Earth)
- $R \sim 6371$ km (radius of the Earth)

Gravitational Force

- **Formulation:**

$$\frac{\mathbf{F}_g}{m} \equiv \mathbf{g}^* = -\frac{GM}{r^2}\left(\frac{\mathbf{r}}{r}\right)$$



$$r = |\mathbf{r}| = R + z \approx R$$

- Force per unit mass exerted on the atmosphere by the gravitational attraction of Earth

$$\mathbf{g}^* = -\frac{GM}{r^2}\left(\frac{\mathbf{r}}{r}\right) \approx -\frac{GM}{R^2}\left(\frac{\mathbf{r}}{r}\right) = \mathbf{g}_0^*$$

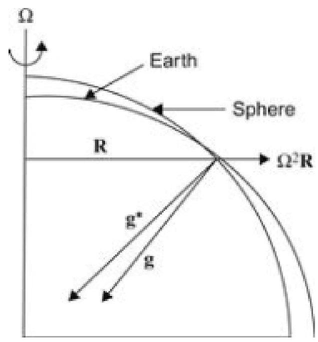
Gravitational Force

- **In a rotating Earth, it is affected by the Earth's angular speed of rotation:**
  - Centripetal acceleration: $-\Omega^2\mathbf{R}$

$$\mathbf{g} \equiv -g\hat{\mathbf{k}} \equiv \mathbf{g}^* + \Omega^2\mathbf{R}$$

  - g~9.81 m s^{-2}
  - Ω~7.292×10^{-5} rad s^{-1}

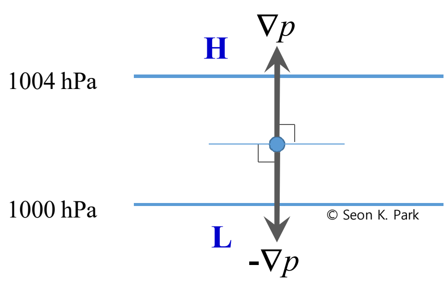
$$\mathbf{F}_g = m\mathbf{g} = -mg\hat{\mathbf{k}} = -\int_V (\rho g)dV\hat{\mathbf{k}}$$



Holton and Hakim (2013)

Pressure Gradient Force

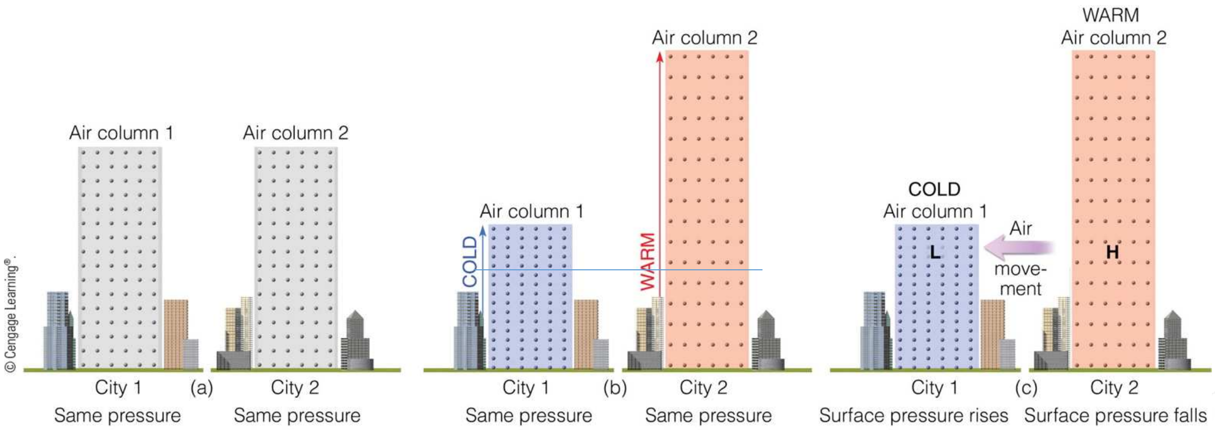
- **Force due to differences in pressure distribution:**
  - Pressure Gradient: ∇p



Note: PGF is in the direction of $-\nabla p$. Why?

Pressure Gradient Force

- **Force due to differences in pressure distribution:**

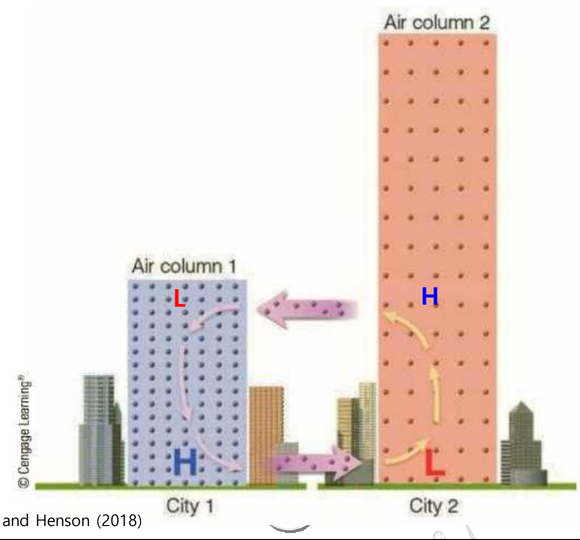


Ahrens and Henson (2018)

Pressure Gradient Force

- **Force due to differences in pressure distribution:**
  - Pressure Gradient: ∇p

The heating and cooling of air columns causes horizontal pressure variations aloft and at the surface. These pressure variations force the air to move from areas of higher pressure toward areas of lower pressure. In conjunction with these horizontal air motions, the air slowly sinks above the surface high and rises above the surface low.



Ahrens and Henson (2018)

Pressure Gradient Force

- Acceleration by PGF ($\mathbf{a}_{PGF}$) – Accelerates an air parcel from a high pressure region to low pressure region, resulting in wind.

$$\mathbf{a}_{PGF} = \left(\frac{d\mathbf{v}}{dt}\right)_{PGF} = -\frac{1}{\rho}\nabla p$$

$\mathbf{v}$: velocity
ρ: air density
p: pressure

- In scalar form,

$$a_x = -\frac{1}{\rho}\frac{\partial p}{\partial x};\quad a_y = -\frac{1}{\rho}\frac{\partial p}{\partial y};\quad a_z = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

Pressure Gradient Force

- Acceleration by PGF ($\mathbf{a}_{PGF}$) – Accelerates an air parcel from a high pressure region to low pressure region, resulting in wind.

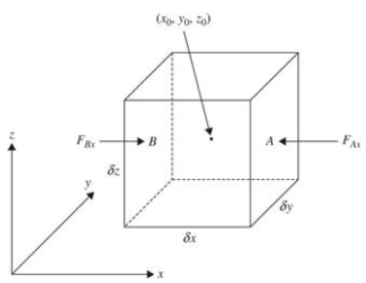
$$(F_A)_x = -\left(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$$

$$(F_B)_x = +\left(p_0 - \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$$

$$(F)_x = (F_A)_x + (F_B)_x = -\frac{\partial p}{\partial x}\delta x \delta y \delta z$$

$m = \rho \delta x \delta y \delta z$

$$\frac{F_x}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x};\ \frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y};\ \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$



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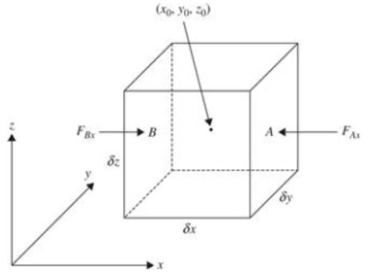
$$\Rightarrow \frac{\mathbf{F}_{PGF}}{m} = -\frac{1}{\rho}\nabla p$$

$$\Rightarrow \mathbf{a}_{PGF} = -\frac{1}{\rho}\nabla p = -\frac{1}{\rho}[(\nabla p)_H + (\nabla p)_z]$$

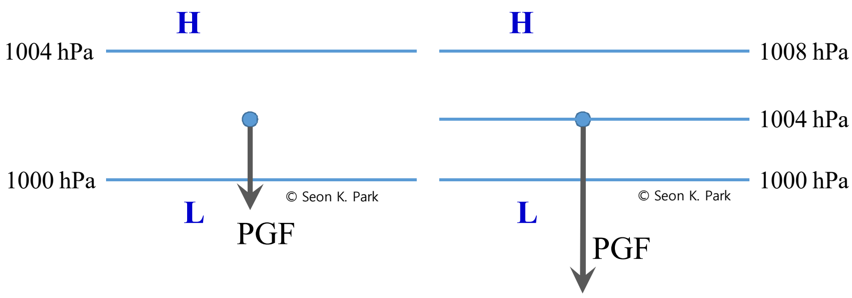


Holton and Hakim (2013)

  - When the vertical term is balanced by gravitational force, atmosphere is in hydrostatic balance.

Pressure Gradient Force

- The greater the pressure difference over a given horizontal distance, the greater the force and hence the stronger the wind.



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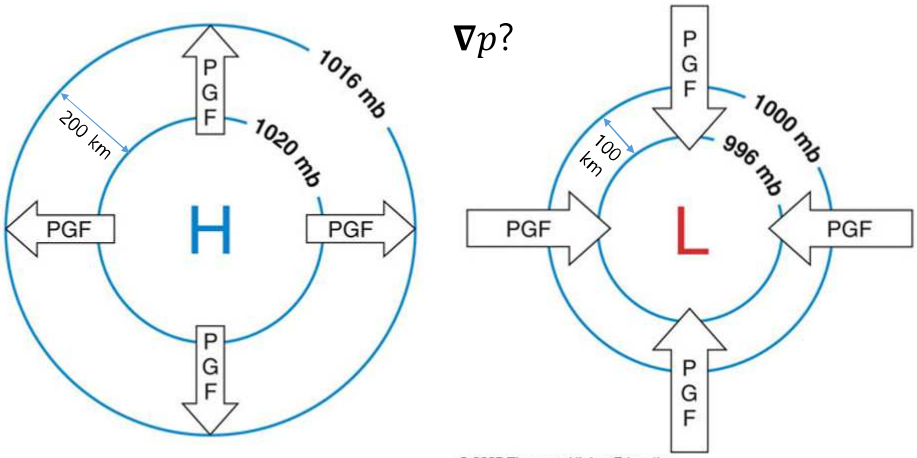

Pressure Gradient Force

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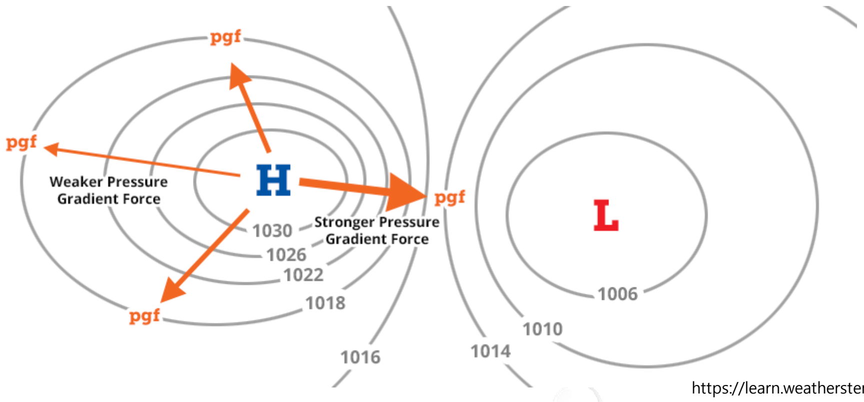
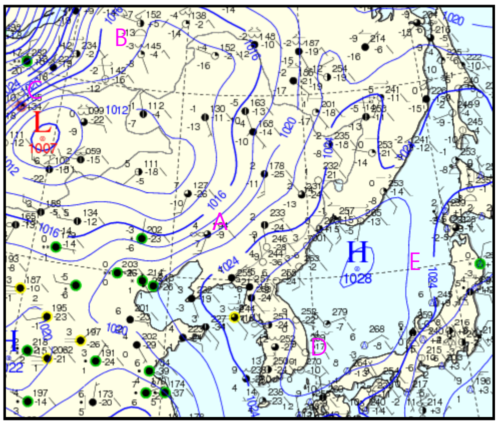


https://learn.weatherstem.com/

Pressure Gradient Force

- Direction and magnitude of PGF in real weather system?



- Direction of PGF?
- The largest PGF?
- The smallest PGF?