TEOREMA PARSEVAL LEZIONE 13

$f(x)$ $\qquad f(x) = \int \tilde{f}(k)\, e^{ikx}\, dk$

$$|f(x)|^2 = f(x)\, f^*(x) = \int \tilde{f}(k_1)\, e^{ik_1 x}\, \tilde{f}^*(k_2)\, e^{-ik_2 x}\, dk_1\, dk_2$$

integro in x

$$\int |f(x)|^2 dx = \iint \tilde{f}(k_1)\, \tilde{f}^*(k_2)\, \underline{\underline{e^{i(k_1-k_2)x}}}\, dk_1\, dk_2\, \underline{dx}$$

$$= 2\pi \int \tilde{f}(k_1)\, \tilde{f}^*(k_2)\, \delta(k_1-k_2)\, dk_1\, dk_2$$

$$= 2\pi \int |\tilde{f}(k)|^2\, dk$$

$$\frac{1}{2\pi} \int |f(x)|^2 dx = \int |\tilde{f}(k)|^2 dk$$

<u>densità spettrale</u>

TEOREMA DI WIENER-KHINCHIN

Funzione di autocorrelazione

$$R(r,x) = \langle f(x+r) f^*(x) \rangle$$

processo omogeneo

$$R(r,x) \to R(r)$$

~~Ass~~ Considero una realizzazione di $f(x)$ in un dominio $[0,L]$

$$R(r) = \langle f(x+r) f^*(x) \rangle = \langle \sum_{k_1} f_{k_1} e^{ik_1 \frac{2\pi}{L}(x+r)} \sum_{k_2} f^*_{k_2} e^{-ik_2 \frac{2\pi}{L} x} \rangle =$$

$$= \sum_{k_1 k_2} \langle f_{k_1} f^*_{k_2} \rangle e^{i(k_1-k_2)\frac{2\pi}{L}x} e^{ik_1 \frac{2\pi}{L} r}$$

integro tra 0 e L in x

$$\cancel{L} R(r) = \cancel{L} \sum_{k_1 k_2} \langle f_{k_1} f^*_{k_2} \rangle \delta_{k_1,k_2} e^{ik_1 \frac{2\pi}{L} r}$$

$$\boxed{R(r) = \sum_{k} \langle |f(k)|^2 \rangle e^{ik \frac{2\pi}{L} r}}$$

Passo al continuo $L \to \infty$ $\Delta k = \frac{2\pi}{L}$ $\Delta k \to 0$ 13,3

$$R(r) = \lim_{\Delta k \to 0} \sum_k \frac{\langle |f(k)|^2 \rangle}{\Delta k} e^{ik\frac{2\pi r}{L}} \Delta k$$

$$\sum_k \ldots \Delta k \to \int dk$$

$$m(k) \underset{\Delta k \to 0}{=} \frac{\langle |f(k)|^2 \rangle}{\Delta k}$$

$$e^{i\frac{2\pi}{L}kr} = e^{i\Delta k k r} \to e^{ikr}$$

$$R(r) = \int m(k) e^{ikr} dk$$

$$\boxed{m(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R(r) e^{-ikr} dr}$$

TEOREMA DI WIENER-KHINCHIN

densità spettrale d'azione

i) LINEARE OMOGENEO $\frac{\partial m(k)}{\partial t} = 0$

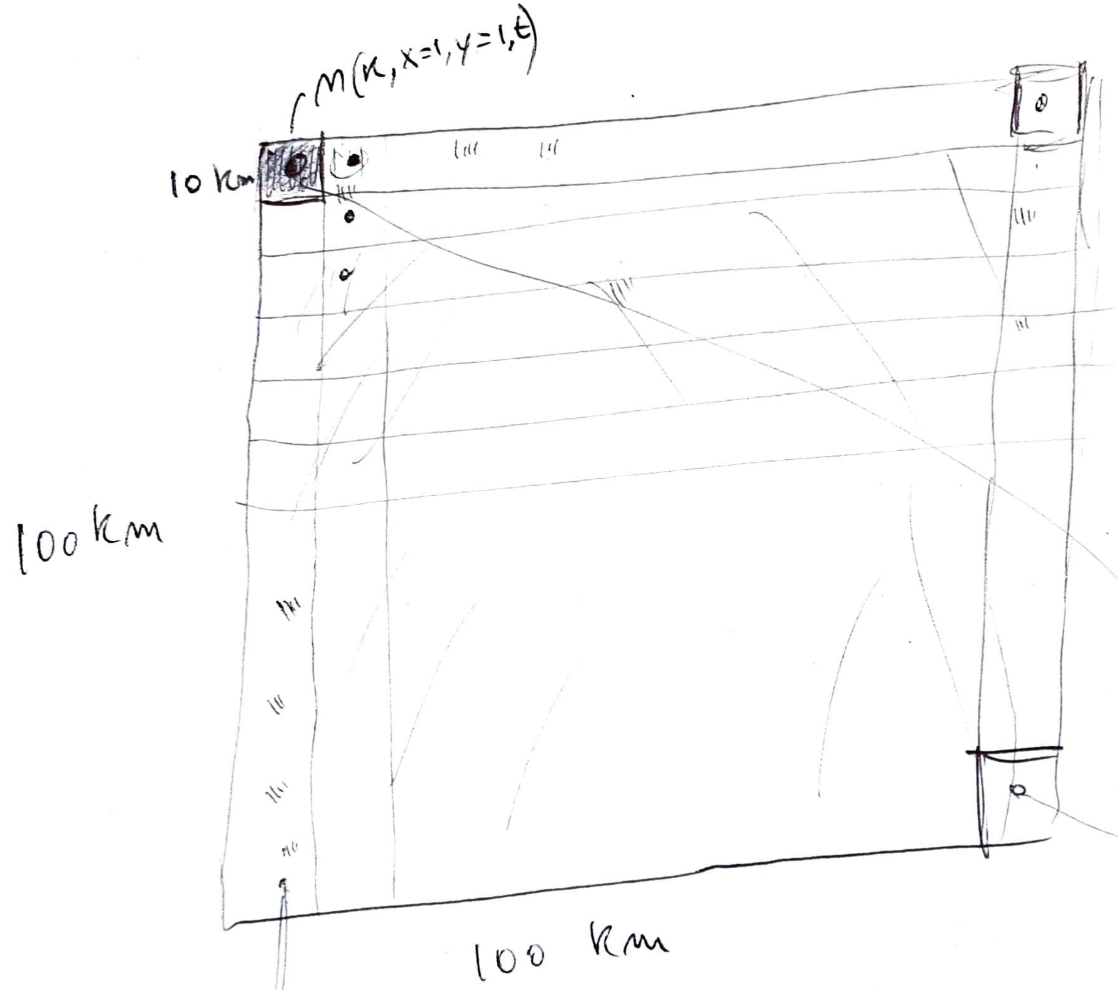
ii) LINEARE MA DEBOLMENTE NON OMOGENEO $m(k, x, t)$ $\boxed{\frac{\partial m_k}{\partial t} + v_{gk} \frac{\partial m}{\partial x} = 0}$

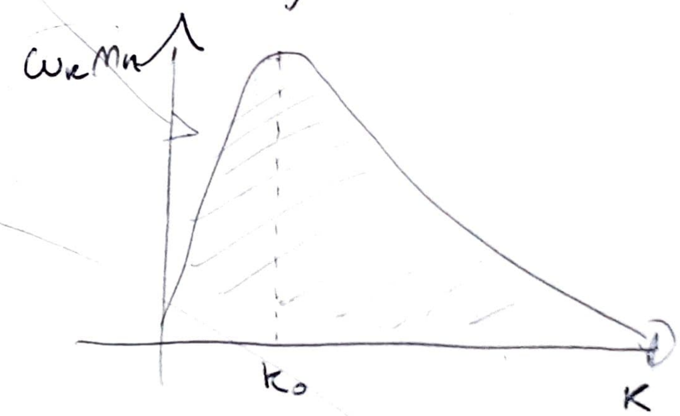
iii) DEBOLMENTE NONLINEARE E DEBOLMENTE NON OMOGENEO

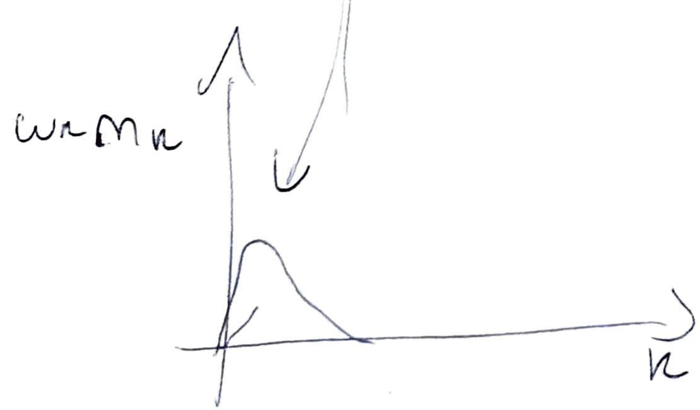
$\to$ EQUAZIONE CINETICA

In ogni sottodominio posso calcolare con l'eq. cinetica $n(k, x, t)$

$$H = \int \omega_k |a_k|^2 \, dk$$







$$\frac{\partial m_{k_1}}{\partial t} + \nabla_{g_k} \cdot \frac{\partial m_{k_1}}{\partial X} = \iint T_{1234}\, m_1 m_2 m_3 m_4 \left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{1}{m_3} - \frac{1}{m_4}\right) \delta(\Delta\omega_{12}^{34})\, \delta(\Delta\vec{k}_{12}^{34})\, d\vec{k}_2 d\vec{k}_3 d\vec{k}_4$$ 13. 2

$$\delta(\Delta\omega_{12}^{34}) = \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \qquad \omega(\vec{k}) = \sqrt{g|\vec{k}|}$$

$$\delta(\Delta\vec{k}_{12}^{34}) = \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4)$$

1 $$\frac{\partial m_{k_1}}{\partial t} = J(k_1) = \underline{\int T_{1234}^2\, G_{1234}\, \delta(\Delta\omega_{12}^{34})\, \delta(\Delta k_{12}^{34})\, dk_{234}} \qquad G_{1234} = m_1 m_2 m_3 m_4 \left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{1}{m_3} - \frac{1}{m_4}\right)$$

INVARIANTI D'URTO

$$\phi(k) \;/\; \int \phi(k) J(k)\, dk = 0 \qquad \frac{\partial}{\partial t}\int m_k \phi_k\, dk = 0 \qquad \int m_k \phi_k\, dk = \underline{\text{cost}}$$

invariante d'urto. $\forall$ invariante d'urto esiste una legge di conservazione

moltiplico 1 per $\phi(k_1)$ e integro in dk_1

$$\int \frac{\partial m_1}{\partial t} \phi_1\, dk_1 = \int \phi_1 T_{1234}^2\, G_{1234}\, \delta(\Delta\omega)\, \delta(\Delta k)\, dk_{234}\, dk_1 = I_1$$

$$\frac{\partial}{\partial t}\int m_1 \phi_1\, dk_1 = I_1 \qquad T_{1234} = T_{2134}$$

$$I_2 = I_1 = \int^{k_1 \leftrightarrow k_2} \phi_2 T_{2134}^2\, G_{2134}\, \delta(\Delta\omega)\, \delta(\Delta k)\, dk_{1234} \qquad G_{1234} = G_{2134}$$

$$I_3 = I_1 = \int \phi_3 T_{3412}^2 G_{3412}\, \delta(\Delta\omega)\, \delta(\Delta k)\, dk_{1234}$$

$k_1 \leftrightarrow k_3$
$k_2 \leftrightarrow k_4$

$G_{3412} = -G_{1234}$

$T_{3412} = T_{1234}$

$$I_4 = I_3 = \int \phi_4 T_{4312}^2 G_{4312}\, \delta(\Delta\omega)\, \delta(\Delta k)\, dk_{1234}$$

$k_3 \leftrightarrow k_4$

$G_{4312} = G_{3412} = -G_{1234}$

$$I_1 = \frac{1}{4}\left(I_1 + I_2 + I_3 + I_4\right) = \frac{1}{4}\int_{-\infty}^{+\infty} (\phi_1 + \phi_2 - \phi_3 - \phi_4)\, T_{1234}^2 G_{1234}\, \delta(\Delta\omega)\, \delta(\Delta k)\, dk_{1234}$$

Quando si annulla l'integrale?

$$\frac{\partial}{\partial t}\int n_k \phi_k\, dk = I_k$$

$\phi_k =$ costante $= 1$ $\Rightarrow$ $N = \int n_k\, dk$ azione totale

$\phi_k = \omega_k$ $\Rightarrow$ $E = \int \omega_k n_k\, dk$ energia

$\phi_k = k$ $\Rightarrow$ $P = \int k\, n_k\, dk$ momento

TEOREMA H ($H \rightarrow$ entropia)

$H = \int \ln(n_k)\, dk \Rightarrow \frac{dH}{dt} \geq 0$

$\frac{dH}{dt} = 0$ all'equilibrio

$$\frac{dH}{dt} = \int \frac{1}{m_{k_1}} \frac{\partial m_{k_1}}{\partial t} dk_1 = \int \frac{1}{m_1} T^2_{1234} \underset{\circledast}{G_{1234}} \delta(\Delta k)\, \delta(\Delta\omega)\, dk_{1234} =$$

13. 4

$$= \frac{1}{4} \int \left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{1}{m_3} - \frac{1}{m_4} \right) T^2_{1234}\, G_{1234}\, \delta(\Delta k)\, \delta(\Delta\omega)\, dk_{1234} = *$$

$$G_{1234} = m_1 m_2 m_3 m_4 \left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{1}{m_3} - \frac{1}{m_4} \right)$$

$$* = \frac{1}{4} \int \left(\frac{1}{m_1} + \frac{1}{m_2} - \frac{1}{m_3} - \frac{1}{m_4} \right)^2 T^2_{1234} \overset{m_1 m_2 m_3 m_4}{\vee} \delta(\Delta k)\, \delta(\Delta\omega)\, dk_{1234} \geq 0$$

Quando $\frac{dH}{dt} = 0$ allora vedi $\circledast$ $\Longrightarrow$ $\frac{1}{m_k}$ è invariante d'urto

$$\frac{1}{m_k} = c_1 + c_2 k + c_3 \omega_k \qquad m_k = \frac{1}{c_1 + c_2 k + c_3 \omega_k} = \frac{1}{c_3} \frac{1}{\left(\frac{c_1}{c_3} + \frac{c_2 k + \omega_k}{c_3} \right)} =$$

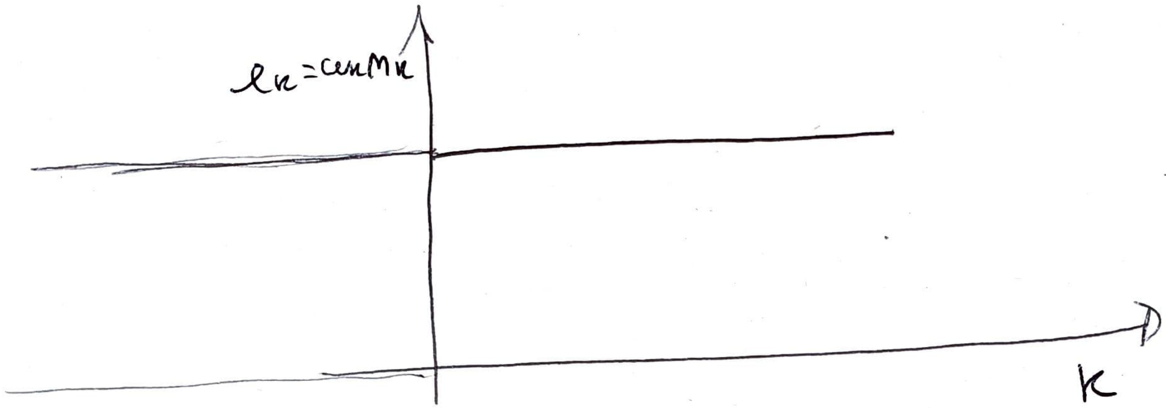
$$\boxed{m_k = \frac{T}{\mu + c k + \omega_k}} \qquad \frac{1}{c_3} = T$$

3 costanti T, μ, c RAYLEIGH - JEANS

Nel caso isotropo $\underline{m(k) = m(-k)}$ $\longrightarrow$ $c = 0$ $\quad m_k = \frac{T}{\mu + \omega_k}$

Se $\mu = 0 \Rightarrow n_k = \frac{T}{\omega_k} \Rightarrow \omega_k n_k = T$ $\quad e_k = T$ $\quad$ lh. 5

densità di energia



equipartizione dell'energia

$$n(k) = n(-k) \qquad n_k = \frac{T}{\mu + ck + \omega_k} \qquad \omega_k = \omega_{-k}$$

$$n_{-k} = \frac{T}{\mu - ck + \omega_k}$$

$$n_k = n_{-k}$$

$$\frac{T}{\mu + ck + \omega_k} = \frac{T}{\mu - ck + \omega_k} \qquad \cancel{\mu} - ck + \cancel{\omega_k} = \cancel{\mu} + ck + \cancel{\omega_k}$$

$$2ck = 0 \qquad c = 0$$