

Mechanisms of precipitation and cloud formation

Textbooks and web sites references for this lecture:

- Robert McIlveen, Fundamentals of Weather and Climate, Chapman & Hall, 1995, ISBN 0-412-41160-1 (§ 6)
- Joseph M. Moran e Michael D. Morgan, Meteorology, The Atmosphere and the Science of Weather, Mc Millan College Publishing Company, 1994, ISBN 0-02-383341-6 (§ 8)

Precipitation processes

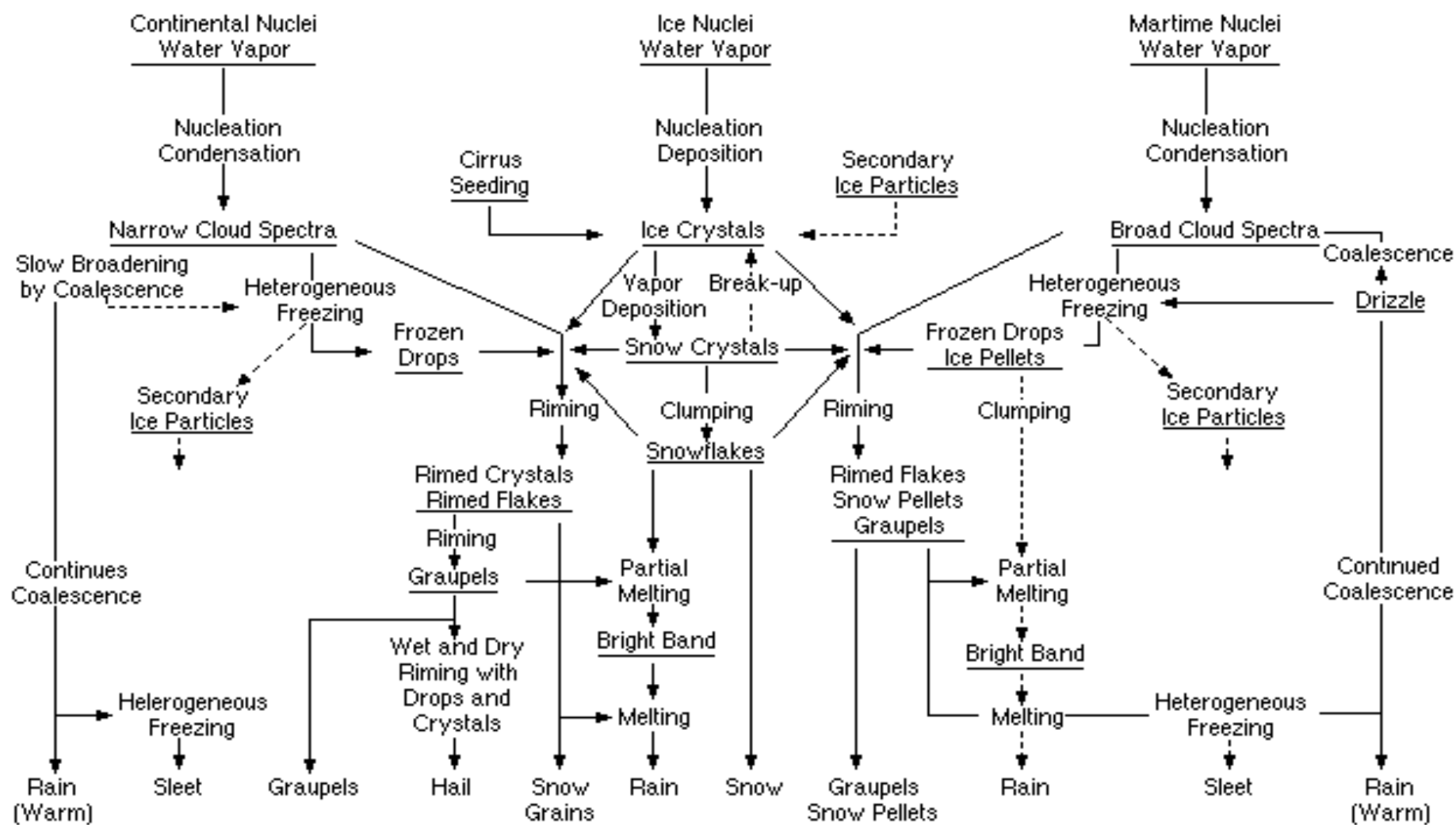
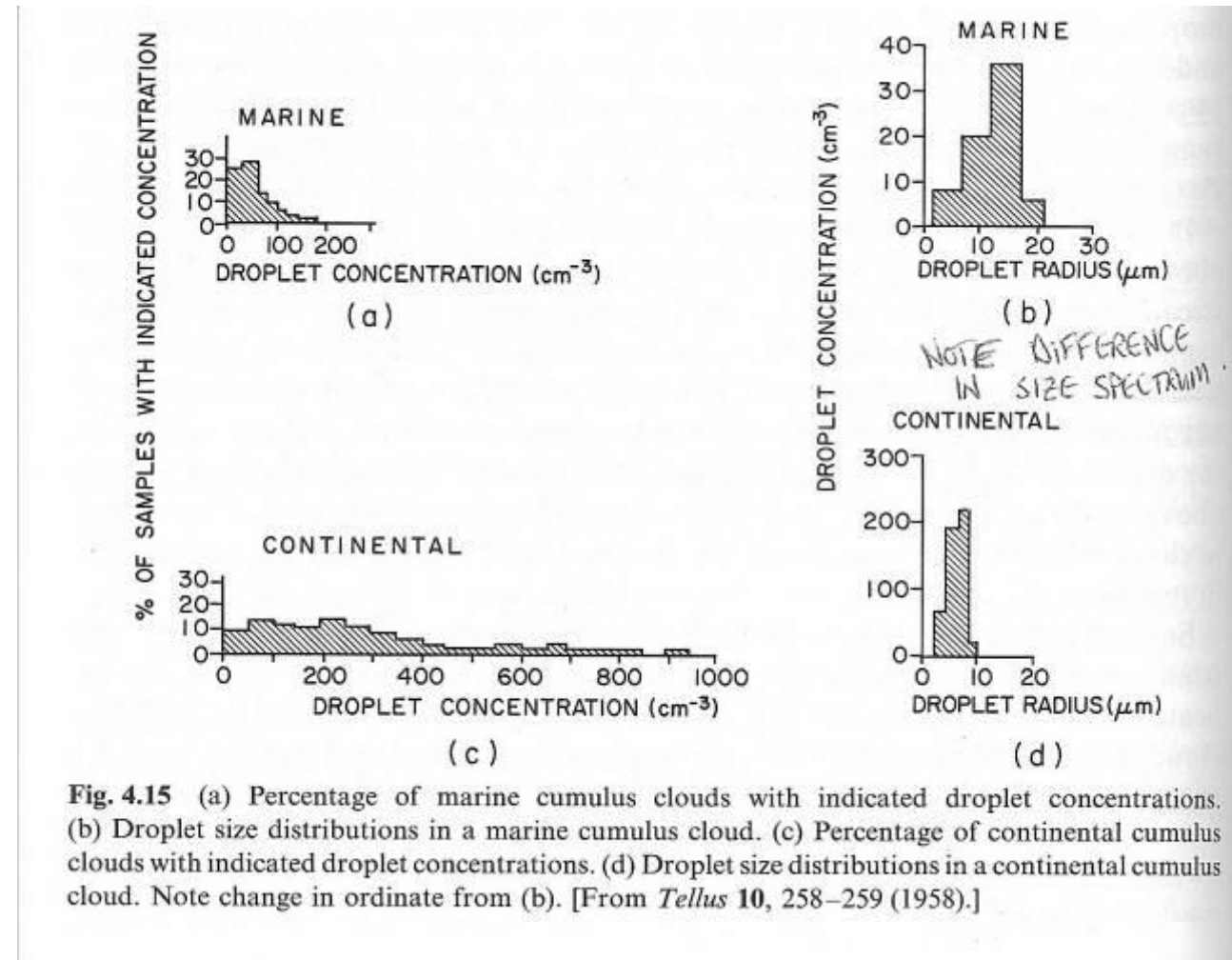


Fig. 1

Effects of Cloud Condensation Nuclei

Results of cumulus clouds grown in marine and continental air



Precipitation types

- **Drizzle**: small droplets; diameter 0.2–0.5 mm, generally precipitating by St or fog
- **Rain**: droplets, diameter > 0.5 mm, precipitating by Ns, Cb or middle-high clouds
- **Freezing rain**: small rain droplets generated in a relatively warm ($>0^{\circ}\text{C}$) layer are precipitating in a cold ($<0^{\circ}\text{C}$) layer becoming supercooled and freezing over cold ($T < 0^{\circ}\text{C}$) surfaces
- **Snow**: formed by single ice crystals or by a conglobation in flakes (sometimes by both); **ice crystals** have a hexagonal structure; snow **flakes** have bigger dimensions when $T \cong 0^{\circ}\text{C}$ (they adhere each other); sometimes ice crystals aggregates forming **ice pellets**, with conic shape, due to growing and collision processes at $T \cong 0^{\circ}\text{C}$, or also **graupel** (snow grains), similar to refrozen rain, or **diamond dust** (precipitation produces by the direct sublimation of atmospheric water vapour at very cold T ; this phenomena can rarely occur also with clear sky)
- **Snow pellets**: frozen drops of diameter < 5 mm, produced by the refreezing of rain droplets when they are falling in a cold air layer sufficiently deep
- **Hail**: precipitation of spherical symmetry or broken ice stones; each “stone” has an internal stratification like an onion, with an alternation of strong (and vitreous) and soft (and opaque) ice layers; produced in the Cb, they do not melt even if T is relatively high because they are big and fall with high velocity

Cloud Parameters

- Macro Scale

- Cloud type
- Cloud amount or cover fraction
- Height and thickness

- Micro scale

- Water content
- Droplet/Crystal size
- Phase

Evaporation and Condensation

Fun facts

- Wind
 - Wind enhances evaporation
- Temperature
 - Warm water evaporates faster than cool water
 - Air temperature affects evaporation rate
 - Cool air, slower molecules, condensation more likely, slows evaporation
 - Warm air can hold more water vapor before saturation than cold air (Clausius-Clapeyron equation)
 - Almost all water vapor is contained in the low troposphere
 - High troposphere is a barrier for the passage of water vapor in the stratosphere

Clouds and precipitations

Fun and experimental facts

- Air masses rise (convective motions) → condensation → clouds
- Experimental observation: air is never oversaturated, even when W is very high: in the clouds $98\% \leq RH \leq 102\%$ → condensation process very rapid
- Water droplets or ice crystals grow until they precipitate
- Clouds cover $>50\%$ of Earth, but precipitations occur only under deep clouds → clouds are very efficient in transferring water from atmosphere to soil → clouds are not too extended
- First 1-2 km layer above soil surface is cloudless (mixing)
- Cloud depth extends from few tenths of m to the whole troposphere
- Clouds are very transitory but cover 50% of the sky → cloud formation and destruction processes are very rapid, and their mechanisms very common

Water residence time

- Above the troposphere there are not any clouds but:
 - o in stratosphere, Cb anvils and nacreous clouds (20-30 km)
 - o in mesosphere, noctilucent clouds (80 km)
- Mean annual rainfall rate: 1000 mm = $5 \cdot 10^{17}$ kg of water
 - o NB: 100 times the consumptions of water in the 2000 year
 - o 1000 mm → 25 mm of rainfall everywhere; $1000/25=40$ precipitations per year; → mean residence time of water in the air: $365/40=10$ days
- Cloud mean depth: 3 km; liquid water content: 0.5 g/m³; mean cloudiness: 50% (but it does not rain below each cloud!)
 - o total cloud liquid water content over the whole Earth: equivalent to 0.8 mm of mean depth; being mean rainfall: 25 mm → cloud LWC: 1/30 of rainfall
- Mean residence time of liquid water in the clouds = 10 days / 30 = 8 h
 - o in truth it varies between 10 min (thunderstorm clouds) and 1-2 days (stratified clouds)
- Deduction: water is present, in the atmosphere, mostly in the cloudless atmosphere

Evaporation and Condensation

- **Evaporation**

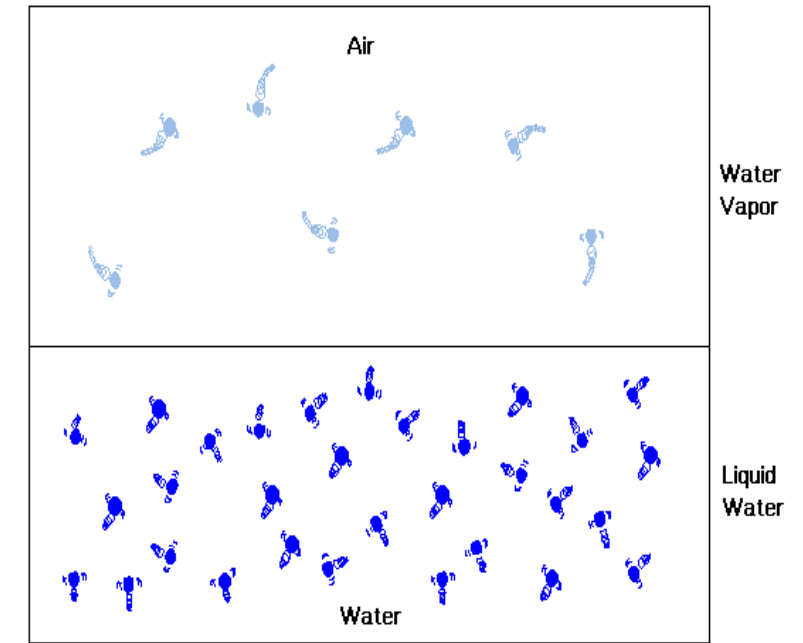
- Fast molecules escape, slower remain: cooling process

- **Condensation**

- Slower molecules collide, form droplets, droplets fall, faster molecules remain: warming process

- **Evaporation/Condensation process transfers heat energy to the atmosphere**

- Latent Heat of Condensation



Changes in state from:
1) Changes in temperature
2) Changes in pressure

Saturation processes

- Dynamical equilibrium in the first mm of air above a **plane wet surface**:
evaporation (or sublimation) \leftrightarrow condensation (or sublimation)

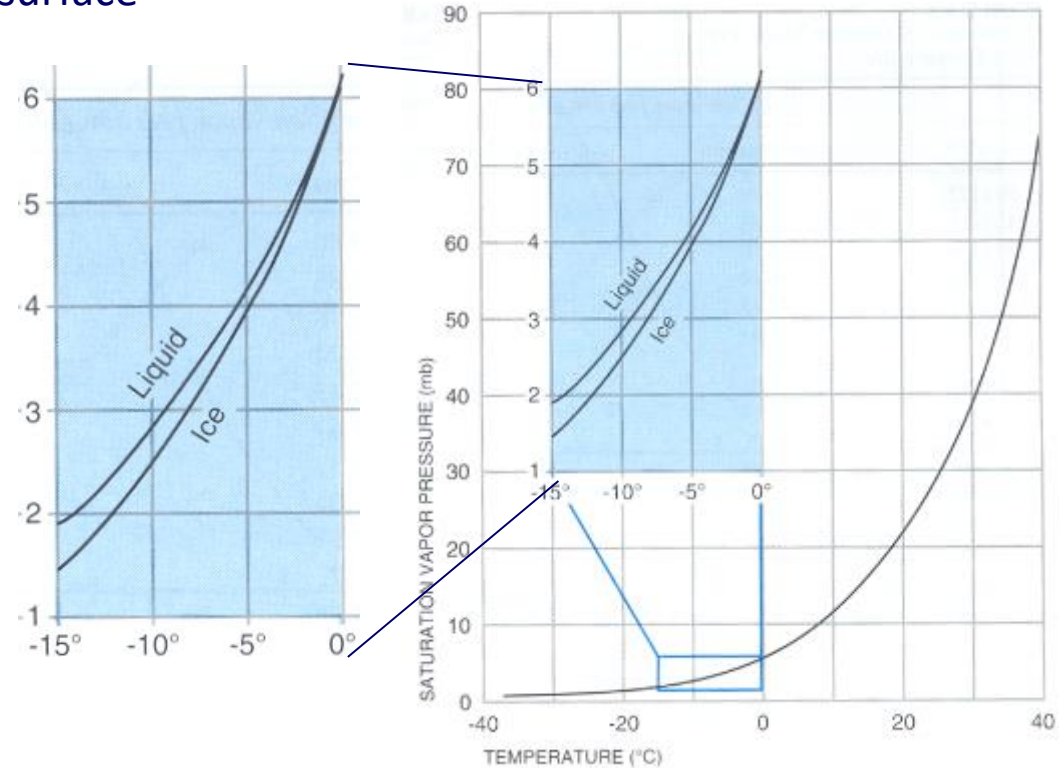
Water vapor (not the air!) is saturated

$$T_{\text{air}} = T_{\text{surface}}$$

- $E_s(T)$ and ρ increase with T
quasi exponentially (Clapeyron)
for pure air

$$\frac{dE_s}{dT} = \frac{L E_s}{R_v T^2}$$

Partial water vapor pressure relative to the ice is lower than that of the water at the same temperature as intramolecular forces in the ice are most intense



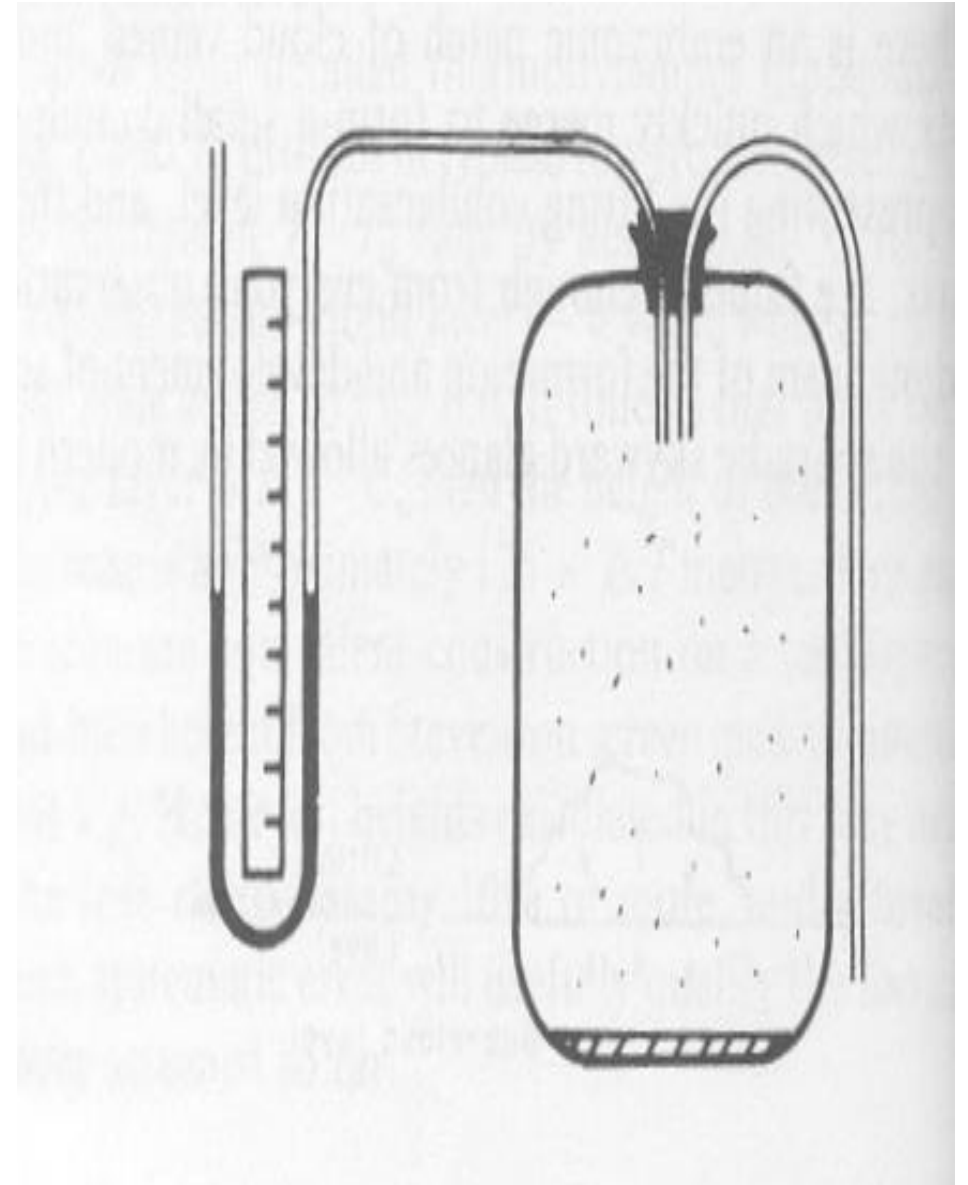
- It is assumed that, in a layer 1 mm deep over a surface covered by ice or water, the water vapor is saturated; at great distances, saturation depends always by the convection

Clouds and cloud formation

- Three requirements for cloud formation:
 1. sufficient moisture in the air to condense; coming from:
 - evaporation (of liquid water)
 - transpiration (of liquid water)
 - sublimation (of ice, snow)
 2. presence of Cloud Condensation Nuclei (CCN)
 3. cooling to cause condensation

Artificial clouds: cloud in a bottle (1/2)

- Need of using large transparent bottles
- Procedure:
 - ❑ put some water in the bottle, then shake it (mixing → same T)
 - ❑ close the extremities of the right pipe with a finger and soufflé in the left pipe until an overpressure for about 30 hPa (30 cm of displacement of the water in the pipe); maintain this overpressure for 30s (causes an increment of T); shake again the bottle
 - ❑ Release the pipe (→ rapid decompression)
- A “thin” cloud will appear (internal air has become oversaturated)
- NB: this joke works only one time! If you try to repeat it at the same conditions, the cloud will result much more thin or inexistent (but it will return dense if we introduce smoke in the bottle)



Artificial clouds: cloud in a bottle (2/2)



A simple experiment to generate a visible cloud inside a plastic bottle.
Watch the movie there:
<https://www.youtube.com/watch?v=wagrbfKV5bE>

What these experiments are teaching us?

- 1) Velocity of cloud droplets growth is extremely rapid (\sim instantaneous) – this is why cloud contours are so net
- 2) CCNs enhance the number of droplets (and thus the cloud density)
- 3) (not so much from my experiment, but well evident from this movie) cloud dissipation is rapid at the same level than cloud formation

The Thomson (Lord Kelvin) law

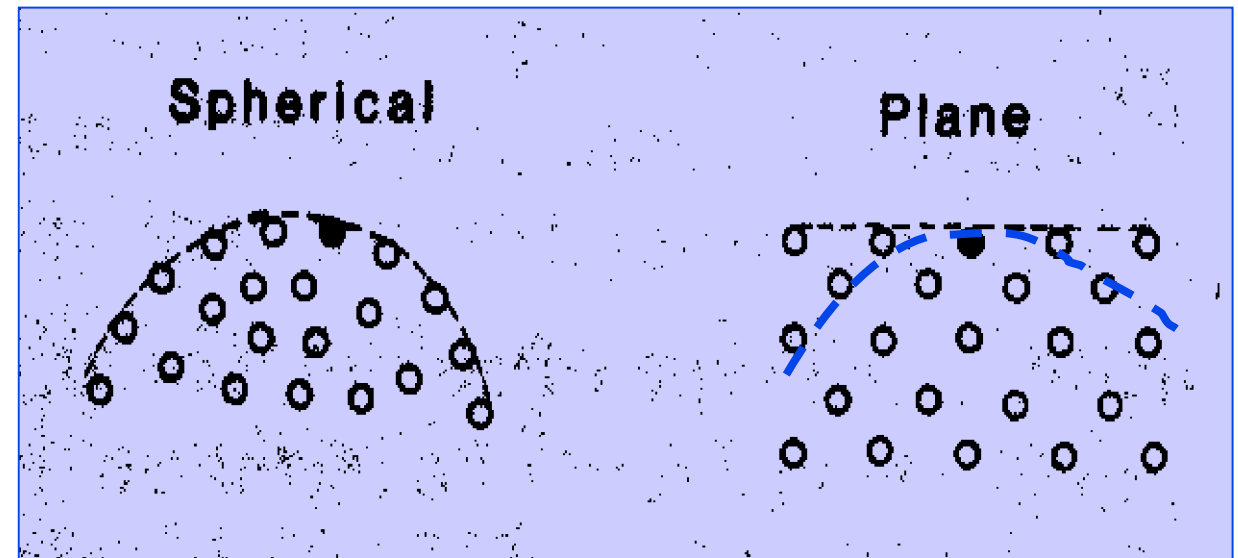
- The atmospheric surfaces (liquid water droplets, water vapor particles, condensation nuclei) are **curve** (~spheres) → very small curvature radii
- The evaporation from a curved surface is **greater** than from a flat surface (there are **fewer** bonds between the molecules → the energy for releasing a molecule is **smaller** → there are **more** molecules becoming free)
- As a result, saturated vapor pressure is **greater** on a curved surface
- The equation of Thomson (Lord Kelvin, 1870) is valid:

A = constant depending by the liquid

e_r = saturated vapor pressure vapor on curved surface with radius r

e_{sat} = saturated vapor pressure on flat surface

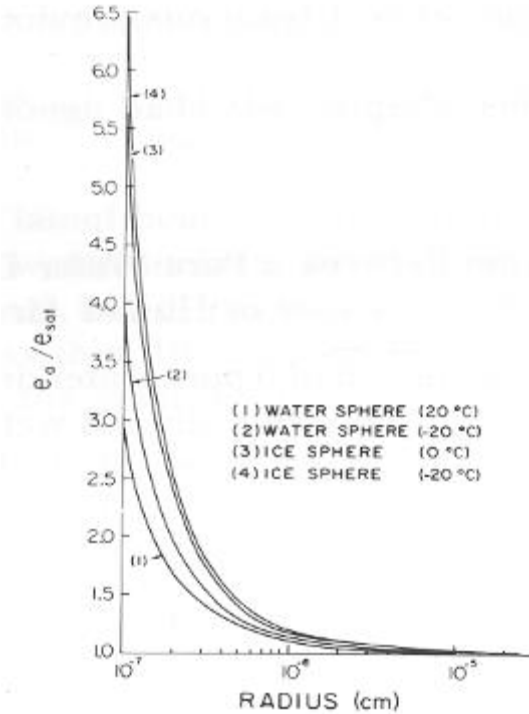
$$\frac{e_r}{e_{sat}} = \exp\left(\frac{A}{r T}\right)$$



The homogeneous nucleation

$$\frac{e_r}{e_{sat}} = \exp\left(\frac{A}{r T}\right)$$

- For $r \approx 10^{-8}$ m (molecular radii) $e_r \gg e_{sat}$ – in this case the droplets are (auto)nuclei (homogeneous nucleation): $e_r/e_{sat}=4 \rightarrow RH \approx 400\%$
- For smaller humidities $e_r < e_{sat} \rightarrow$ evaporation > condensation \rightarrow even if droplets form, they re/evaporate immediately \rightarrow no condensation
- In atmosphere so high humidities (400%) are never measured \rightarrow **the homogeneous nucleation is a very rare and inefficient process** to form droplets into the atmosphere
- But if the particles are larger ($0.15 \mu\text{m}$), then $e_r/e_{sat}=1.01$ ($\rightarrow RH=101\%$) \rightarrow **the homogeneous nucleation is efficient ONLY for large particles**

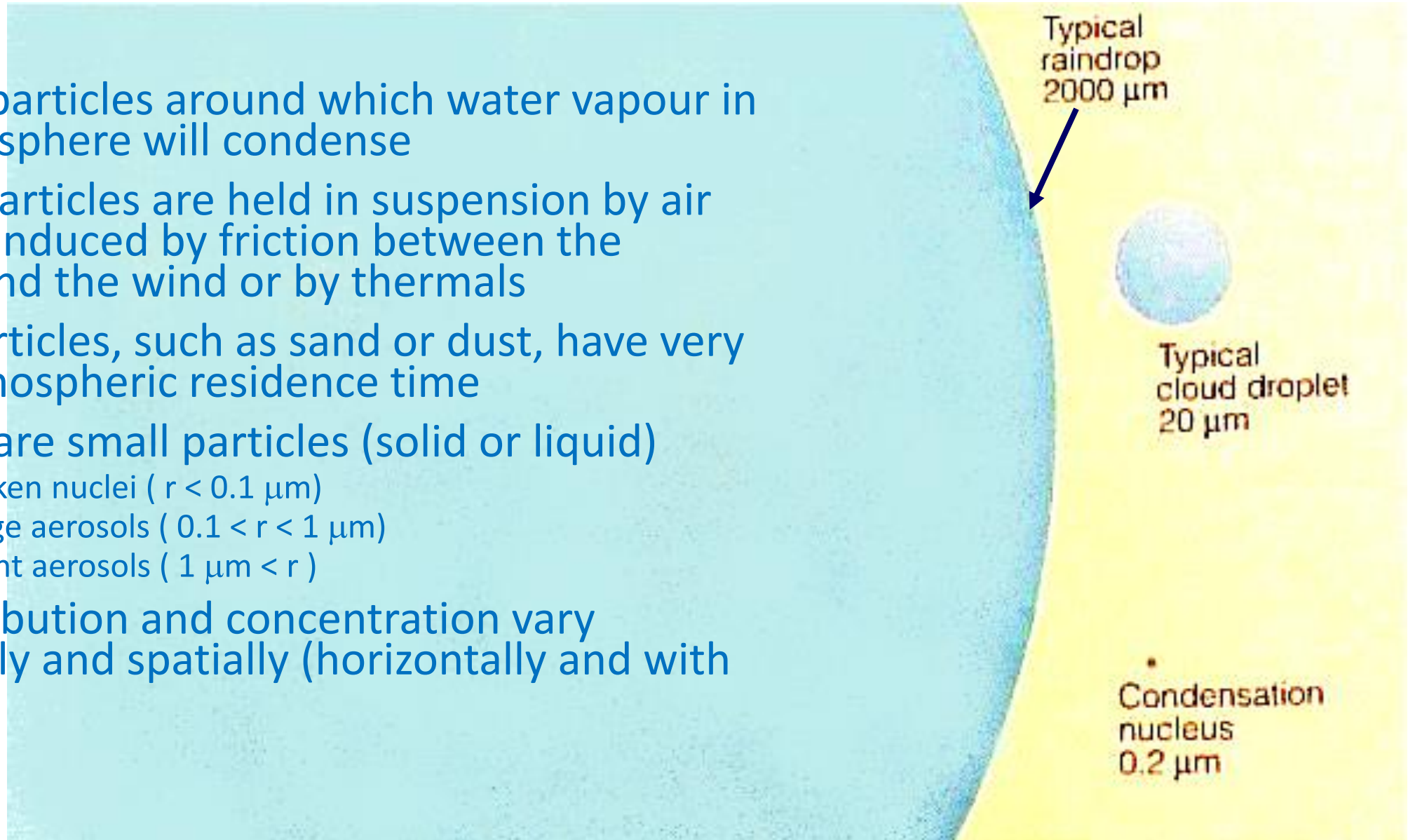


Formation of Cloud Droplets

- presence of CCN in the atmosphere provides a potential for water to condense out of the vapor phase, given saturated conditions
- 2 factors regarding CCN ability to condense out vapor:
 - » water condenses easier on larger aerosols due to vapour pressure
 - » many aerosols are hygroscopic (attract water onto a surface)
 - » condensation can thus occur even if air is not fully saturated with water
- water droplet density in the order of 10^9 m^{-3} (3 to 4 orders of magnitude less than CCN concentration)

Cloud Condensation Nuclei (CCN)

- CCN are particles around which water vapour in the atmosphere will condense
- smaller particles are held in suspension by air currents induced by friction between the ground and the wind or by thermals
- larger particles, such as sand or dust, have very short atmospheric residence time
- aerosols are small particles (solid or liquid)
 - Aitken nuclei ($r < 0.1 \mu\text{m}$)
 - large aerosols ($0.1 < r < 1 \mu\text{m}$)
 - giant aerosols ($1 \mu\text{m} < r$)
- size distribution and concentration vary temporally and spatially (horizontally and with height)



Why are there CCNs?

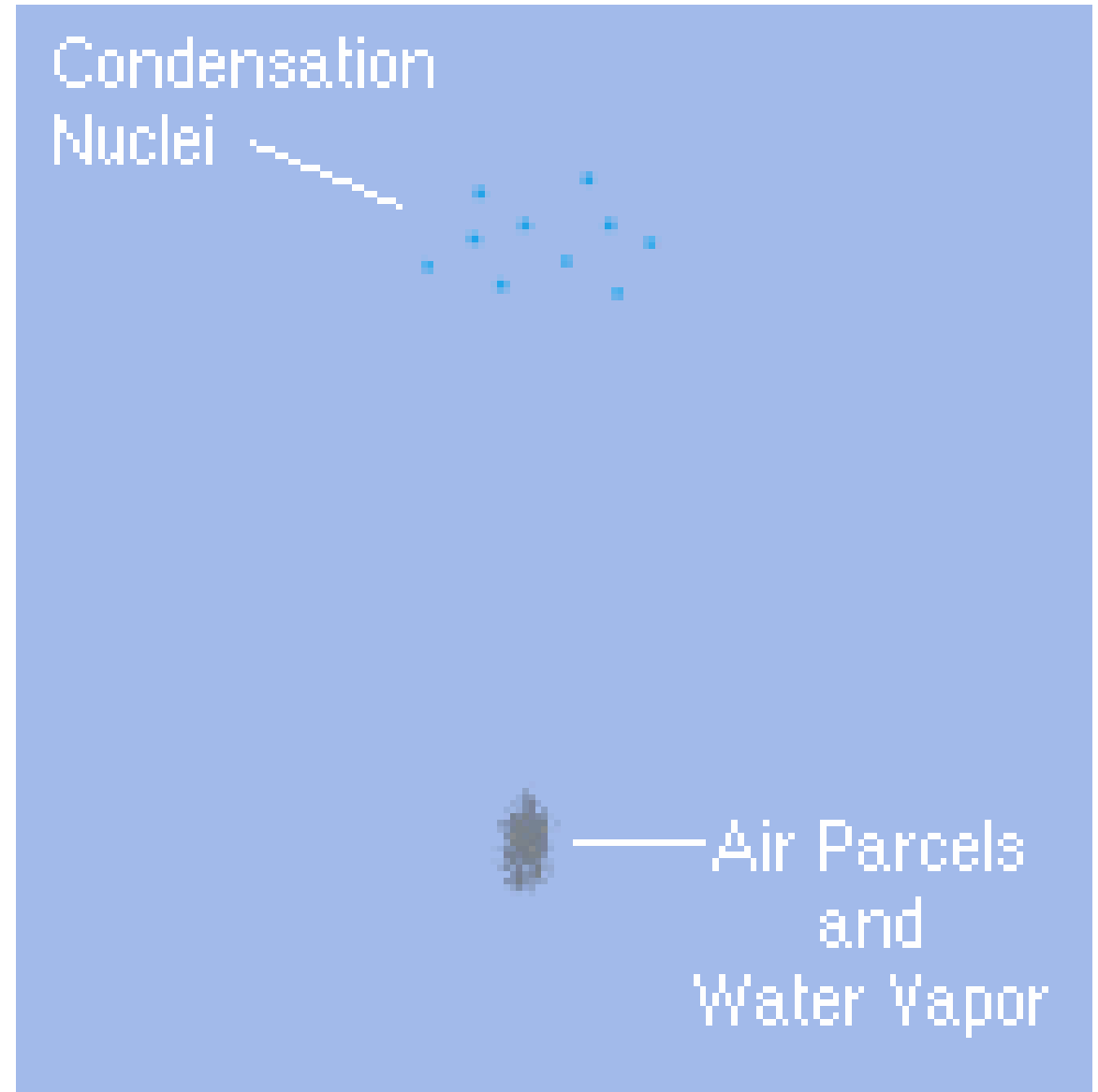
- CCN concentrations are typically in the order of 10^{12} m^{-3}
- sources of CCN may be natural or anthropogenic
- concentrations may increase by up to 2 orders of magnitude over and downwind of industrial areas (eg. St Louis, MO generates CCN at a rate of $10^{-2} \text{ m}^{-3} \cdot \text{s}^{-1}$)

Worldwide aerosol production in tonnes per annum (*after Wallace & Hobbs, 1971*)

Natural sources	10^6 tpa	Human activities	10^6 tpa
Sea salt	1000	Gas-to-particle conversion	275
Gas-to-particle conversion	570	Industrial processes	56
Windblown dust	500	Fuel combustion	
Forest fires	35	(stationary sources)	44
Meteoric debris	20	Solid waste disposal	2.5
Volcanoes (highly variable)	25	Transportation	2.5
TOTAL	>2150	Miscellaneous	28
		TOTAL	410

Process of Cloud Formation

- ❑ CCN
 - Dust
 - Salt Particles from Sea Spray
 - Natural Aerosols
 - Human Created Pollution
- ❑ Air rises and cools to saturation - most effective nuclei are activated
- ❑ Saturation vapor pressure decrease as parcel continues to rise and cool - the parcel becomes supersaturated
- ❑ More CCN activate at a higher humidity



Condensation for heterogeneous nucleation

- Many droplets are made not only by pure water but by solutions, sometimes even very dense (especially hygroscopic particles, such as NaCl) with radii $\leq 1 \mu\text{m}$
- Many of these substances are hygroscopic solids \rightarrow a solution forms \rightarrow the saturation equilibrium is different.
- In this case, Köhler law applies:

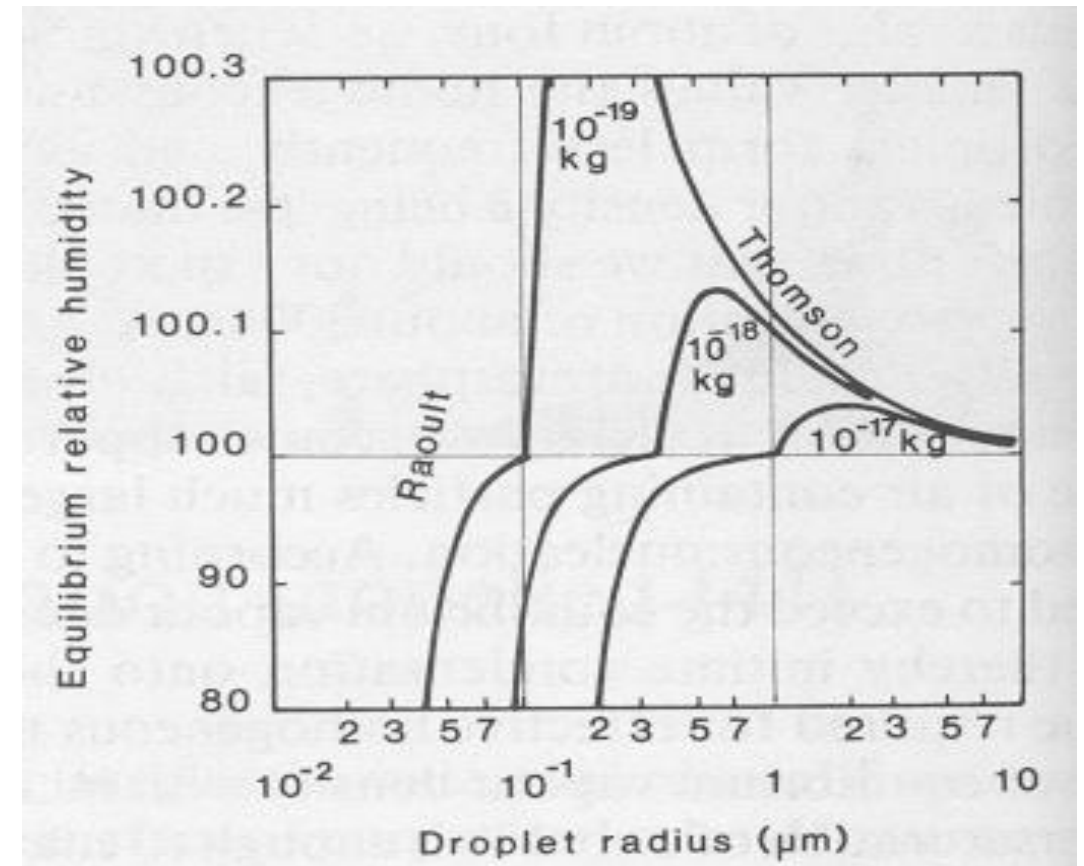
$$\text{Soluble} \quad \frac{e_r}{e_{sat}} = 1 + \frac{A}{r T} - \frac{Bm}{r^3}$$

$$\text{insoluble} \quad \frac{e_r}{e_{sat}} = 1 + \frac{A}{r T} - \frac{B'm}{r^3 - r_u^3}$$

B = constant depending by the solute type

m = solute mass dissolved in the droplet

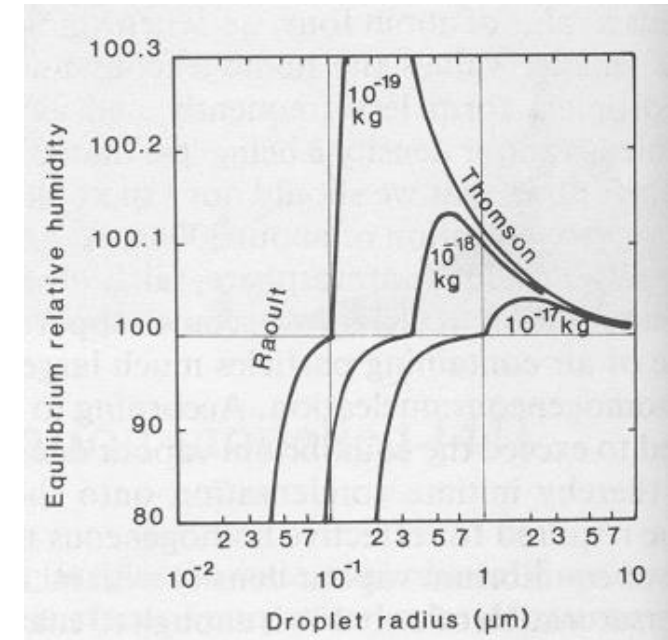
r_u = radius of insoluble part



The Köhler law

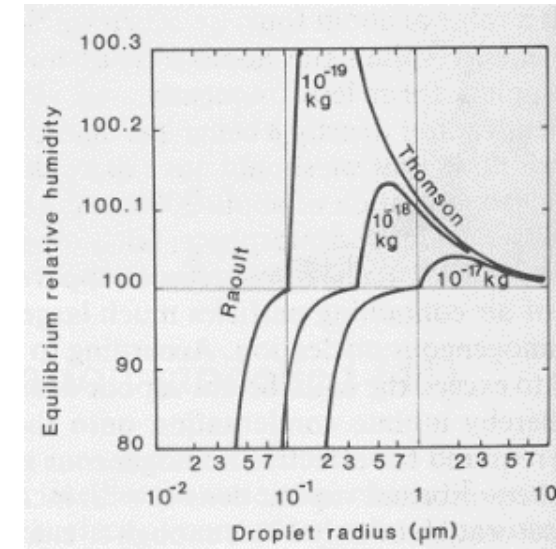
$$\frac{e_r}{e_{sat}} = 1 + \frac{A}{rT} - \frac{Bm}{r^3}$$

- First two terms are the development in Taylor series (truncated at the 1st order) of the Thomson equation
- When small droplets of a solution of NaCl (or any other solution) are in equilibrium with the supersaturated vapor, the 3rd term dominates → there are droplets also with $e_r/e_{sat} \leq 1$
- For very large radii, the Thomson effect (first 2 terms) dominates
 - → the supersaturation reduces to zero
- For intermediate values of the radius, **there is a critical radius $r_c = (3BmT/A)^{0.5}$ for which the supersaturation is maximum**: r_c depends on m and T



The Köhler law

$$\frac{e_r}{e_{sat}} = 1 + \frac{A}{r T} - \frac{Bm}{r^3}$$



- For $r < r_c$ the droplet is in **stable** equilibrium: if by chance the cloud droplet loses (earns) some molecules, its radius decreases (increases) to r' so that $e_r' < (>) e_r$ and then, being $e_r' < (>) e$, a vapor flow creates, which produces r' increase (decrease) back to the same r
- For $r > r_c$ the droplets of a solution is in **unstable** equilibrium: if by chance the cloud droplet loses (earns) some molecules, its radius increases (decreases) to r' so that $e_r' > (<) e_r$ and then, being $e_r' > (<) e$, a vapor flow creates, which produces r' decrease (increase), so that it cannot return to the same r value
- For $e = e_r$ such that $e_r/e_{sat} > 1$, $r = r_c$: thus this supersaturation acts and **the cloud activates**

Growth for diffusion and condensation

- A rising aerosol particle of mass m and radius r expands and cools adiabatically. As T decreases, its RH (in dynamic equilibrium with the surrounding air) increases. When $RH \leq 100\%$ a layer of concentrated solution forms
- If the particle continues to rise, its RH grows approaching 100%. For the Raoult law, also the radius of the solution droplet r ($< r_c$) grows for diffusion and condensation. Water vapor is in dynamic equilibrium
 - a particle of NaCl grows from $r_1=0.1 \text{ mm}$ [$RH_1=87\%$] to $r_2=0.2 \text{ mm}$ [$RH_2=99\%$])
- This equilibrium persists until $r \leq r_c$, i.e. the atmosphere is slightly supersaturated ($100 \leq RH \leq 101\%$) (for example, 100.13% for NaCl)
- If RH grows further, $r \geq r_c$: there are no values of r for which evaporation > condensation \rightarrow growth without limits (ACTIVATION) occurs at “explosive speed”: this process is called **growth for diffusion and condensation**
- For drops with $r \approx 1 \mu\text{m}$ visibility is drastically reduced. Also growth process is reversible. Thus boundaries in convective clouds are net

Cloud development

Nuclei contained in a dm ³ of air		
Aitken nuclei	0.005 – 0.1 μm	10 ⁷
Big nuclei	0.1 – 1.0 μm	10 ⁵
Giant nuclei	> 1.0 μm	10 ³

Nuclei with $r < 0.005 \mu\text{m}$ never activate

Nuclei are formed by SO₂, salt, soil particles, volcanic particles, combustion residuals, anthropic products

When the air mass rises:

- At $\text{RH} \cong 80\%$ the condensation begins; at $\text{RH} \cong 90\%$ $r \leq 1 \mu\text{m} \rightarrow$ faint haze
- At $(100 \leq \text{RH} \leq 101)$ $r > r_c \rightarrow$ cloud activation first on giant nuclei, then on big ones, then on Aitken ones
- Growth is quite fast (a few s) up to $r_c \cong 1 \mu\text{m} \rightarrow$ due to the sunlight scattering, the cloud has net boundaries also with quite high W
- Inverse process (deactivation) is just as quick as the direct one \rightarrow reversible
- Initially, number of droplets = number of active nuclei ($\leq 10^7 \text{ l}^{-1}$)
- The size of the drops depend (also) from the available water vapor

Growth of Cloud Droplets

- as droplets grow initial increase in radius is rapid
- for larger particles, the increase in radius (with increasing surface area) is much lower
- as clouds age, drop size decreases since larger droplets break due to air motion
- all droplets are subject to the force of gravity
 - in a stable, undisturbed environment all droplets fall
 - fall rate increases until the frictional force (F_D) and the gravity force are equal - terminal velocity is reached
- in real clouds
 - uplift causes smaller particles to remain in suspension
 - larger particles still fall against upcurrents
 - small particles falling into an unsaturated environment often dissipate due to large evaporation surfaces

Water vapor flux

- The flux F ($=\Delta m_v/S\Delta t$, kg/m²s) of vapor through a droplet of radius r can be expressed using the **Fick law**: $F=D \, de/dr$:

$$\frac{\Delta m_v}{\Delta t} = 4\pi r^2 D \frac{de}{dr}$$

D =coefficient of diffusion of vapor in air

de/dr =vapor pressure gradient from the center of the droplet

- Integrating on n and on the density, we obtain:

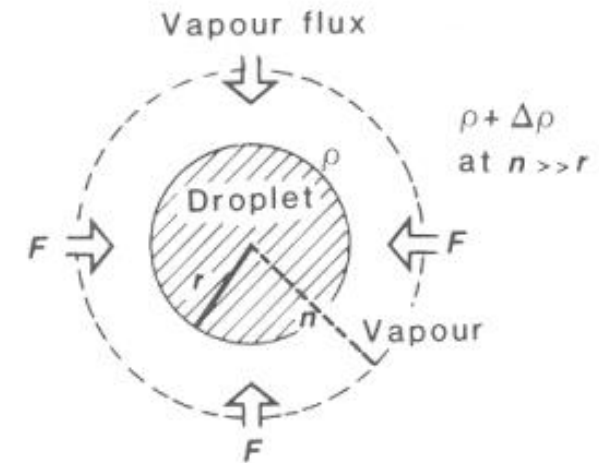
$$\int_r^\infty \frac{\Delta m_v}{\Delta t} \frac{dr}{r^2} = \int_{e_{sat}}^{e_r} 4\pi D de$$

- In stationary conditions, the flux do not vary:
- If $\Delta e \neq 0$, there is a mass (=density x volume) variation :

$$\frac{\Delta m_v}{\Delta t} = \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \rho_w \right) \text{ from which}$$

$$\frac{\Delta m_v}{\Delta t} = 4\pi r D (e_r - e_{sat})$$

$$r \frac{dr}{dt} = D \frac{e_r - e_{sat}}{\rho_w}$$



Droplet growth law

- The difference of vapor pressure Δe between the saturated (e_{sat}) surface of drop and the supersaturated ($e_a > e_{sat}$) external environment can be expressed as:

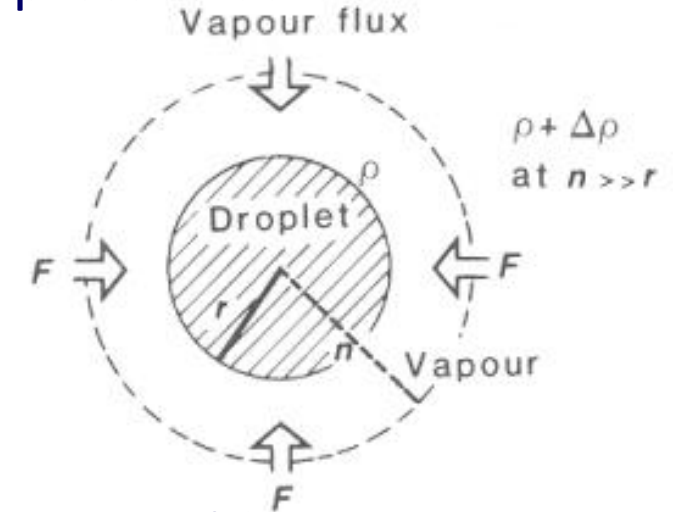
$$\Delta e = e_r - e_{sat} = e_{sat} \left(\frac{e_r}{e_{sat}} - 1 \right) \cong \frac{e_{sat}}{100} (RH - 100)$$

- If $\Delta e = \text{const}$, then $r^2 \propto t$, i.e. we get:

$$r \propto \sqrt{t}$$

$$t = \frac{100 \rho_w}{2 D e_{sat}} (r_2^2 - r_1^2)$$

- The growth is initially very rapid, but then decreases quickly: it would never cause rain! To arrive to $r = 0.1 \text{ mm}$ more than 2 hours are needed!



R (μm)	1	2	4	8	16	32	64
T (s) doubling radius	0.9	4.3	18	72	291 ≅ 5 min	1163 ≅ 19 min	4608 ≅ 77 min

Falling speed of droplets (1)

- A spherical body of radius r and density ρ_w immersed in air (density ρ) in free fall is in a state of balance between:

- Weight force
$$F_P = mg = \frac{4}{3} \pi r^3 g \rho_w$$

- Buoyancy (Archimedes) force
$$F_G = m_a g = \frac{4}{3} \pi r^3 g \rho$$

- Friction force
$$F_A$$

$$F_A = F_P - F_G = \frac{4}{3} \pi r^3 g (\rho_w - \rho) \cong \frac{4}{3} \pi r^3 g \rho_w$$

Falling speed of droplets (2)

- For $r < 30 \mu\text{m}$ (cloud droplets, aerosol) the speed is very low \rightarrow laminar motion

$$F_A = 6\pi \rho \nu V r \qquad V = Y_1 r^2, \quad Y_1 = \frac{2g\rho_w}{9\nu\rho} = 1.2 \cdot 10^8 \text{ (ms)}^{-1}$$

- For $30 \mu\text{m} \leq r \leq 1 \text{ mm}$ (big cloud droplets, rain droplets) the motion is turbulent:

$$F_A = K V r^2 \qquad V = Y_2 r, \quad Y_2 = \frac{4\pi g \rho_w}{3K} = 8 \cdot 10^3 \text{ s}^{-1}$$

- For $r > 1 \text{ mm}$ (big rain droplets, hailstones) the motion is very turbulent:

$$F_A = K' V^2 r^2 \qquad V = Y_3 \sqrt{r}, \quad Y_3 = \sqrt{\frac{4\pi g \rho_w}{3K'}} = 250 \text{ m}^{0.5} \text{ s}^{-1}$$

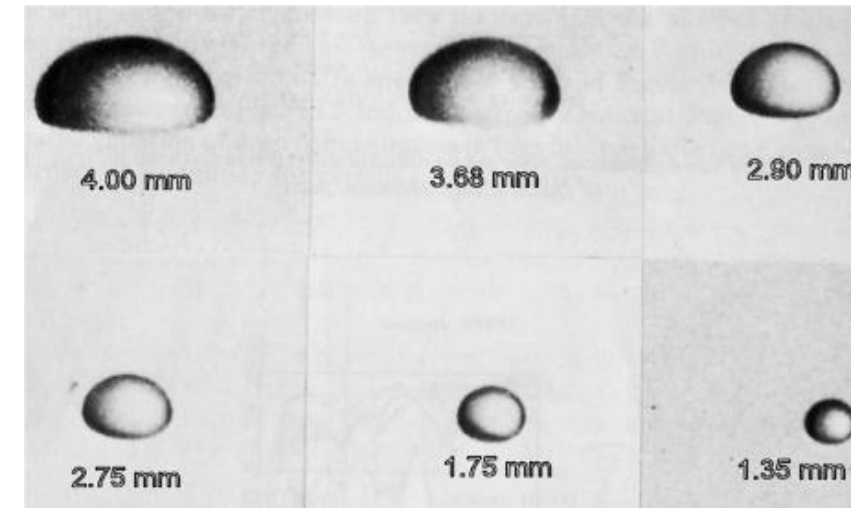
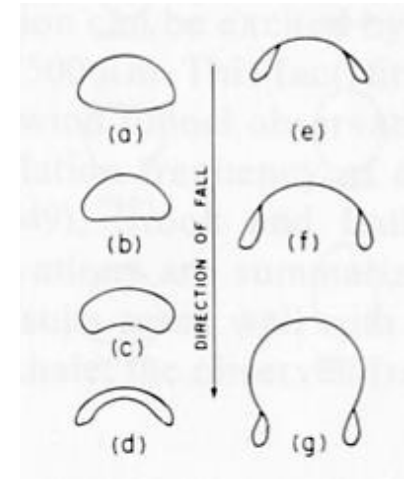
NB: big rain droplets can be broken due to the friction, hailstones more rarely

Examples of fall speed values

		Diameter	Fall speed (m/s)	Form
rain	drizzle	0.2 mm	0.8	Spherical
	Light rain	0.5 mm	4.0	Spherical
	Strong rain	5.0 mm	10.0	Variable unstable
cloud	Small droplets	1 μm	10^{-4}	Spherical
	droplets	10 μm	0.01	Spherical
snow	Small cristals	0.2 mm	0.3	prism
	Medim cristals	5.0 mm	0.7	star
	Small flakes	1.0 mm	0.5	Irregular
	Big flakes	20.0 mm	1.0	Irregular
hail	Graupeln	0.5 mm	0.5	Conic
	Light hail	5.0 mm	2.5	Conic
	hail	3.0 mm	8.0	Spherical
	Giant hail	> 20.0 mm	20.0	Spherical irregular

Shape of raindrops

- Raindrops have a size-dependent shape which cannot be characterized by a single length. Raindrops are described in terms of equivalent diameter, defined as the diameter of the sphere of the same volume as the deformed drop
- When falling at terminal velocity, drops are nearly perfect spheres if $d > 280 \mu\text{m}$
- Larger drops are slightly deformed and resemble oblate spheroids if $280 < d < 1000 \mu\text{m}$
- For $1 < d < 10 \text{ mm}$ the deformation becomes large and the drops resemble oblate spheroids with flat bases
- Drops with $d > 10 \text{ mm}$ are hydrodynamically unstable and break up



Raindrop spectrum

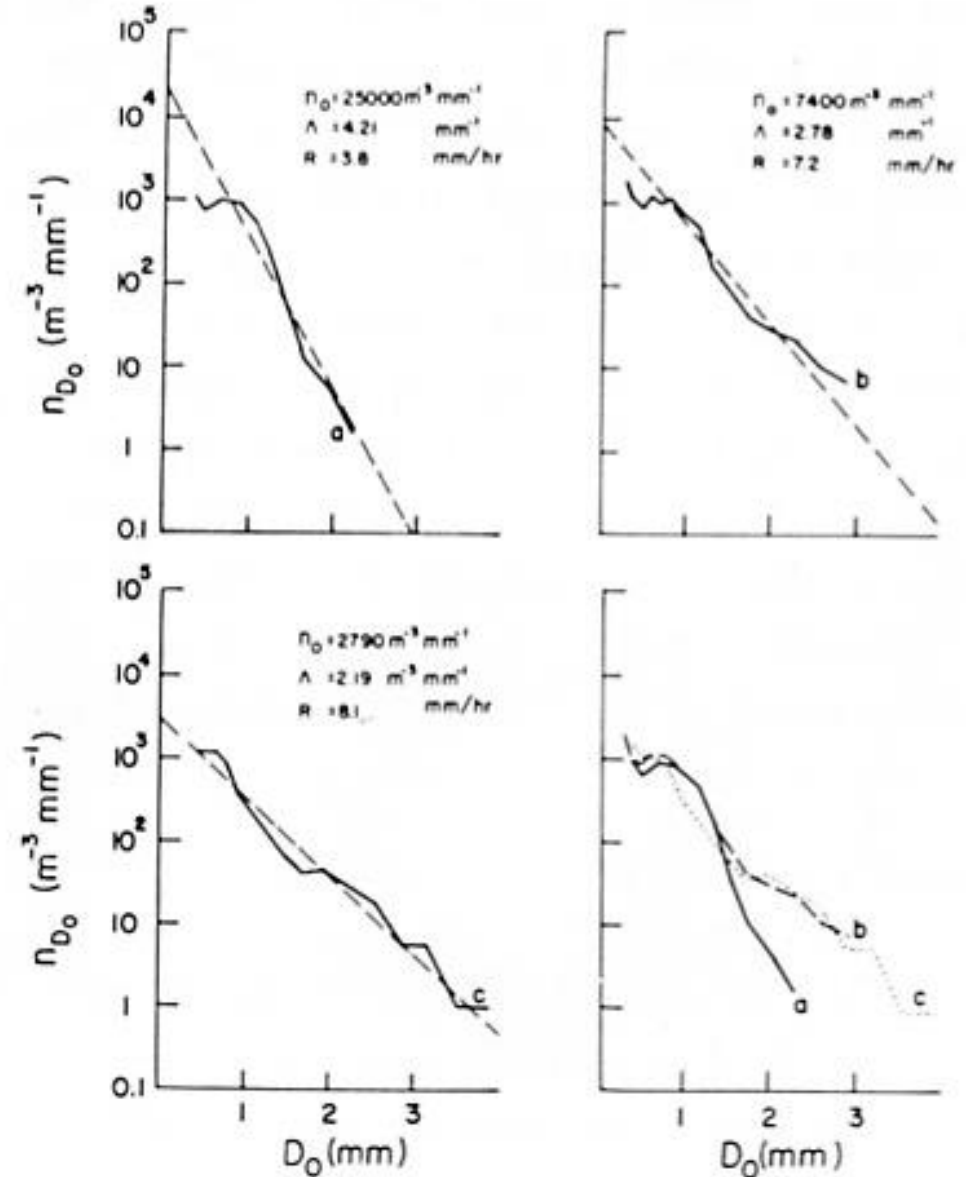
- Raindrop spectra may extend to drop diameters as large as 6 mm, but such large drops are rather rare since they are found only in very heavy rain with $R > 100$ mm/h; at smaller rainfall intensities raindrop spectra usually extend to diameters of 2-3 mm
- Destructive factors: turbulence, fragmentation due to collisions, evaporation
- Constructive factors: collection of smaller drops
- Usually, near the beginning of a rainstorm, the drop spectrum at ground level may be expected to be biased toward large sizes owing to the greater fall speeds of the larger drop, and possibly toward small sizes owing to an initially high evaporation rate
- The true spectrum is quite complicated, and in part determined by temperature, relative humidity and wind in the subcloud region

Raindrop size distribution

- The most widely used empirical equation to describe the raindrop size distribution is that of Marshall and Palmer (1948):

$$n(D_0) = n_0 e^{-\Lambda D_0}$$

- where D_0 is the diameter of raindrop (mm), n_0 and Λ are empirical parameters depending on site and type of precipitation



Evaporation of cloud and rain droplets

- The growth of clouds is balanced by precipitation
- Precipitation types depend on water droplet diameter (100 μm - 3 mm)
- The lower limit of the droplets is given by the re-evaporation speed
- The space traveled is given by $dz = V dt = V (dt / dr) dr$;
- For small cloud droplets ($r < 30 \mu\text{m}$)

$$t = \frac{100 \rho_w}{2(RH - 100) De_{sat}} (r_2^2 - r_1^2)$$

- If $V = Y_1 r^2$, assuming $RH = \text{const}$, we get $z = K r^4$ with:

$$K = \frac{100 \rho_w}{(RH - 100) De_{sat}}$$

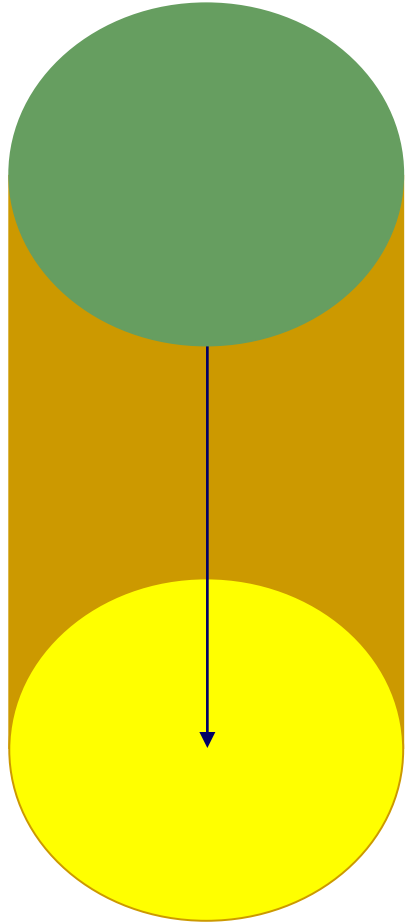
- Assuming realistic values ($RH = 95\%$), the result is that a droplet of 1, 10, 100, 1000 μm respectively travel 2.7 μm , 2.7 cm, 270 m, 2700 km before evaporating completely \rightarrow small droplets evaporate before reaching the ground

Collision and Coalescence

- The growth process by diffusion and condensation is too slow for $r > 5 \mu\text{m}$; moreover, after a certain time from the formation of the first droplets, there is a wide spectrum of droplet sizes, and therefore speeds
- The biggest drops incorporate or attract the smaller ones (**growth by collision and coalescence**). Simplified models of precipitation from clouds for collisions have been developed. The real problem is, of course, very complex: some smaller drops can circumvent, or bounce, the biggest ones. The collision efficiency is variable, and very low for $r < 20 \mu\text{m}$
- A larger drop falls faster than a smaller drop
- The larger particle overtakes, possibly collides with, and potentially coalesces (fuses) with the smaller droplet
- Hocking (1959) showed that the drop minimum radius is $19 \mu\text{m}$ (via condensation) in order than collisions with smaller droplets occur
- Collision efficiency increases as size of the colliding drop increases; broad droplet spectra favor more collisions
- Not all collisions result in coalescence: other processes act, like turbulence, surface contaminants, electric fields and charges



Times of growth per collision of a small raindrop



- In the simplest case, a “**small**” rain droplet of radius r falls at constant speed V , spacing the volume $\pi r^2 V dt$ in the time dt
- If the specific humidity of the cloud droplets is q , as they can be considered approximatively at rest with respect to the raindrop, their density is $q\rho$ and their mass is $q\rho \pi r^2 V dt$

- The raindrop in the time dt collects the mass $(\pi r^2 V dt)(q\rho)$ and changes its mass by:

$$\Delta m = \rho_w \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \right) \text{ from which we get: } \frac{dr}{dt} = \frac{q\rho}{4\rho_w} V$$

- When $r < 30 \mu\text{m}$ the falling speed is $V=Y_1 r^2$: $\frac{dr}{dt} = \frac{q\rho Y_1}{4\rho_w} r^2 = \frac{r^2}{A}$
 $A=3.3 \cdot 10^{-2} \text{ ms if } q=1 \text{ g/Kg}$

- Integrating on time, we get:

$$\Delta t = A \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

i.e. $\Delta t \approx 14 \text{ min for } 20 \mu\text{m} \rightarrow 40 \mu\text{m}$

(and time halves for each doubling of r)

- In these conditions, to pass from 30 to $300 \mu\text{m}$ 19 minutes are needed

Times of growth per collision of a medium raindrop

- If the falling raindrop has **medium dimensions** ($30\text{ }\mu\text{m} < r < 1\text{ mm}$), its fall speed is $V=Y_2\text{ }r$, thus we get:

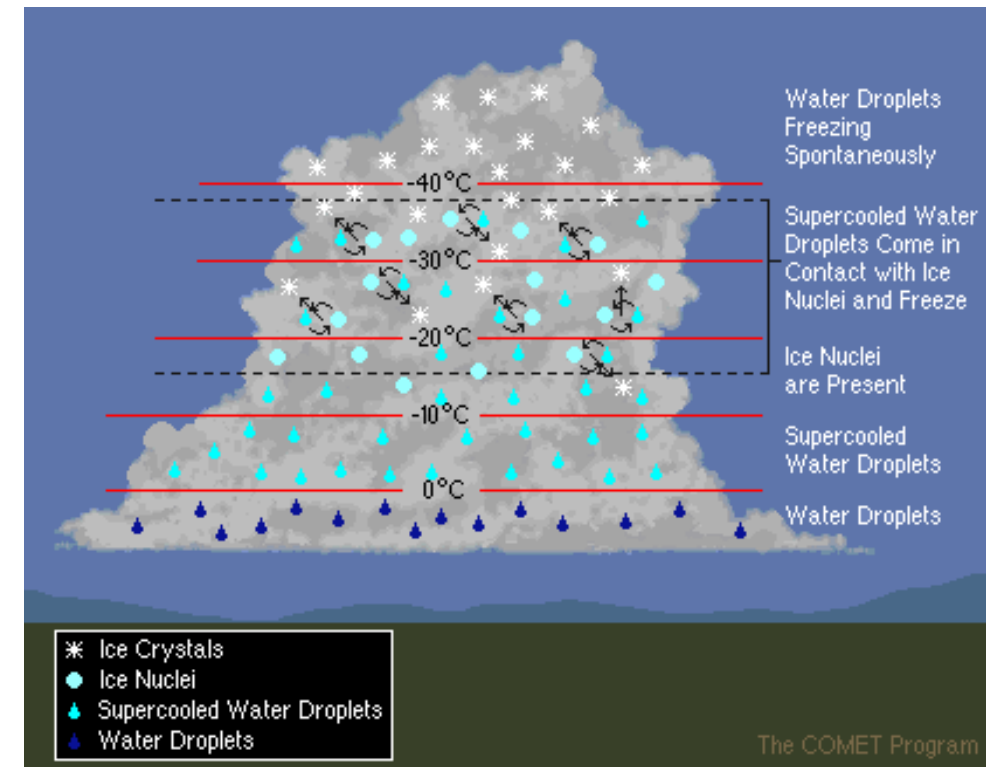
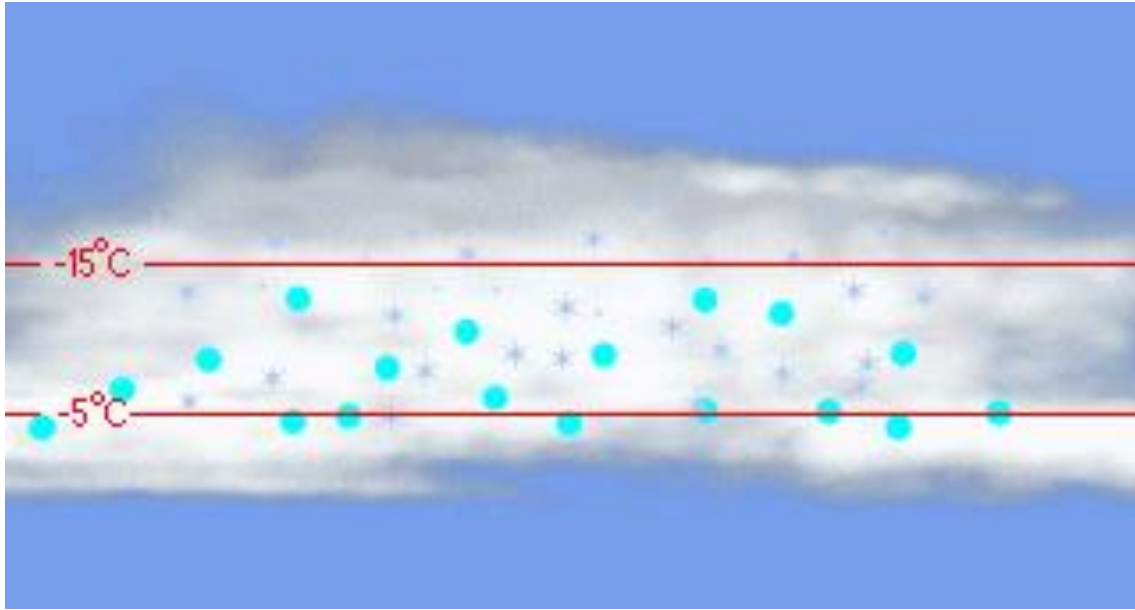
$$\frac{dr}{dt} = \frac{r}{B}$$

$$B = \frac{4\rho_w}{q\rho Y_2} = 500\text{ }s$$

$$\Delta t = B \ln \frac{r_2}{r_1}$$

Small raindrop	Medium raindrop	Initial size
$\Delta t\text{ } (r_2 = 2\text{ }r_1)\text{ } r_1 < 10\text{ }\mu\text{m}$	$\Delta t\text{ } (r_2 = 2\text{ }r_1)\text{ } r_1 < 10\text{ }\mu\text{m}$	$r_1\text{ }(\mu\text{m})$
5.8 min		0.1
5.8 min		1
5.8 min		2
5.8 min		5
5.8 min		10
5.8 min		20
	5.5 min	50
	2.8 min	100
	33 s	500
	16.5 s	1000

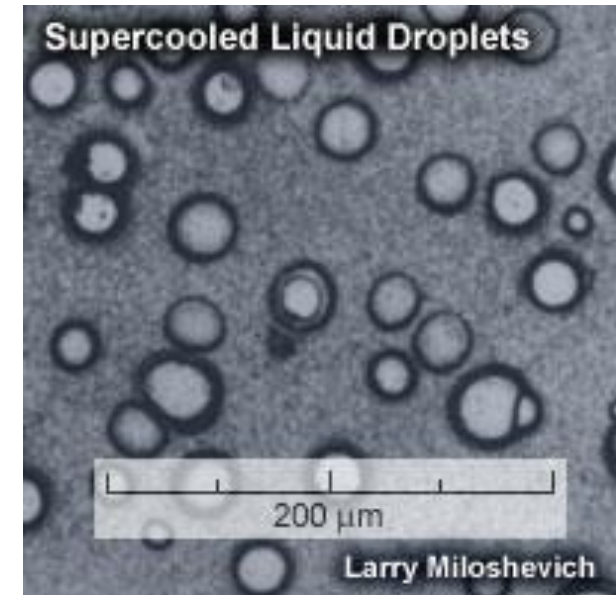
Ice Crystal Growth in Clouds



- Supercooled water can exist in a liquid state between -40 and 0°C
- Cloud type based on temperature
 - **WARM** clouds contain only water droplets ($T > 0^{\circ}\text{C}$)
 - **MIXED** clouds contain supercooled water and ice (above -12°C supercooled water dominates due to hostility of cloud environment to the freezing process)
 - **COLD** clouds contain only ice particles

Mixed clouds

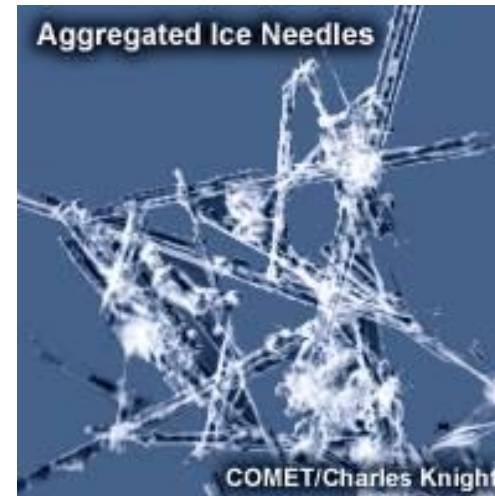
- For $-20 \leq T \leq 0^\circ\text{C}$ ice particles coexist with supercooled water drops; the former grow at the expense of the latter



- Mechanisms:
 - Water vapour diffusion to the ice particles (deposition)
 - Drop collision with and freezing on the ice particles (riming)
 - Originated as a frozen drop or ice crystal and has grown by riming to an irregular or roundish, semitransparent particle
 - Frozen drops or partially melted and subsequently refrozen snow crystals or snowflakes

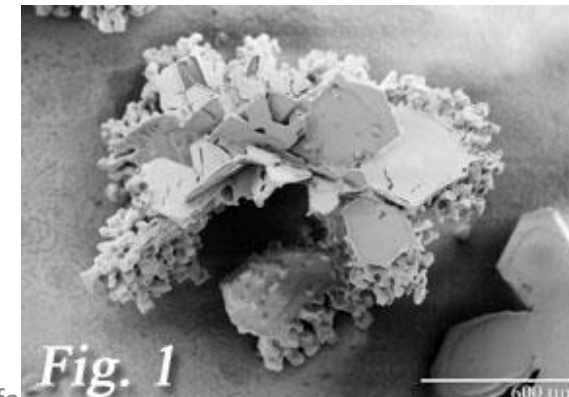
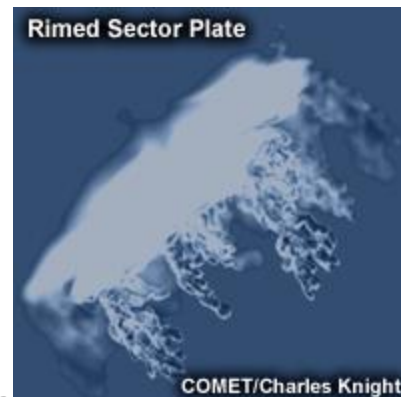
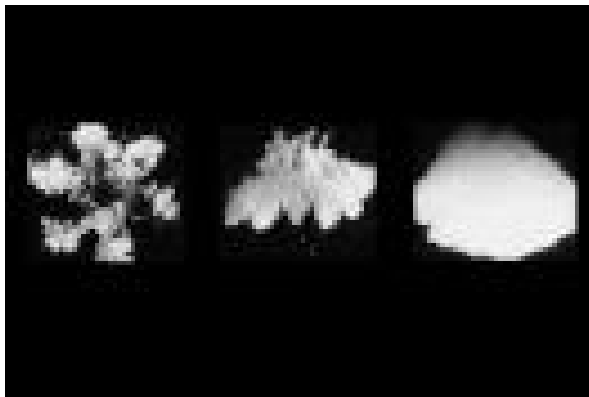
Deposition

- Water vapour diffusion to the ice particles (deposition)
- Ice or snow crystals ($d < 5$ mm) are produced
- they can grow:
 - by collisions with other crystals (clumping)
 - By aggregation with other crystals (snow flakes, $d < 2$ cm)



Riming

- Drop collision with and freezing on the ice particles (riming):
- In the initial phase of riming, as long as the features of the original ice crystal are still well distinguishable, the ice particle is simply called a lightly or densely rimed snow crystal
- When riming of an ice particle has proceeded to the stage where the features of the primary ice particle are only faintly or no longer visible, the ice particle is called a graupel particle, a soft hail particle or a snow pellet
- Such a particle has a white, opaque and fluffy appearance due to the presence of a large number of air capillaries in the ice structure. It usually has a bulk density of less than 800 kg/m^3



Riming

- In the later stage of riming, such particles may have a conical, rounded or irregular shape.
- Normally these particles have $d < 5$ mm; when their dimensions are $d > 5$ mm, they are called hailstones; they have roundish, ellipsoidal or conical shape often with obes or other protuberances on the surface; they are opaque and in cross section exhibit an onion-type layered structure with alternating opaque and clear layers caused by presence of more or less numerous air bubbles



Small hail particles, type-b ice pellets

- Originates as a frozen drop or ice crystal and subsequently grows by riming to an irregular or roundish, semitransparent particle (with or without a conical tip)
- In this case it is called a small-hail particle, or type-b ice-pellet, and its bulk density is higher ($800\text{-}900\text{ kg/m}^3$) as this particle may contain water in its capillary system

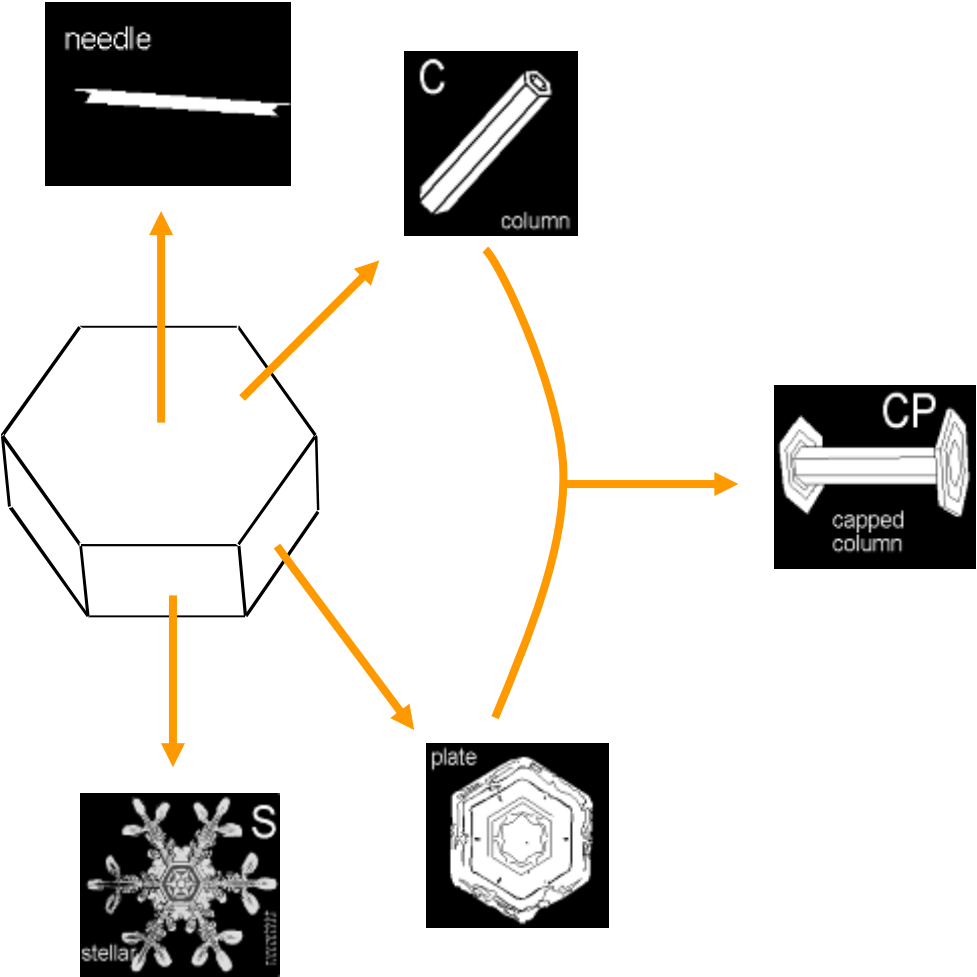


Sleet, type-a ice pellets

- Frozen drops or partially melted and subsequently refrozen snow crystals or snowflakes are hard, transparent and of globular or irregular shape
- Their bulk density is $\sim 990 \text{ kg/m}^3$ and they are called type-a ice pellets or sleet, and can contain unfrozen water



Particle Shape – Crystal classification



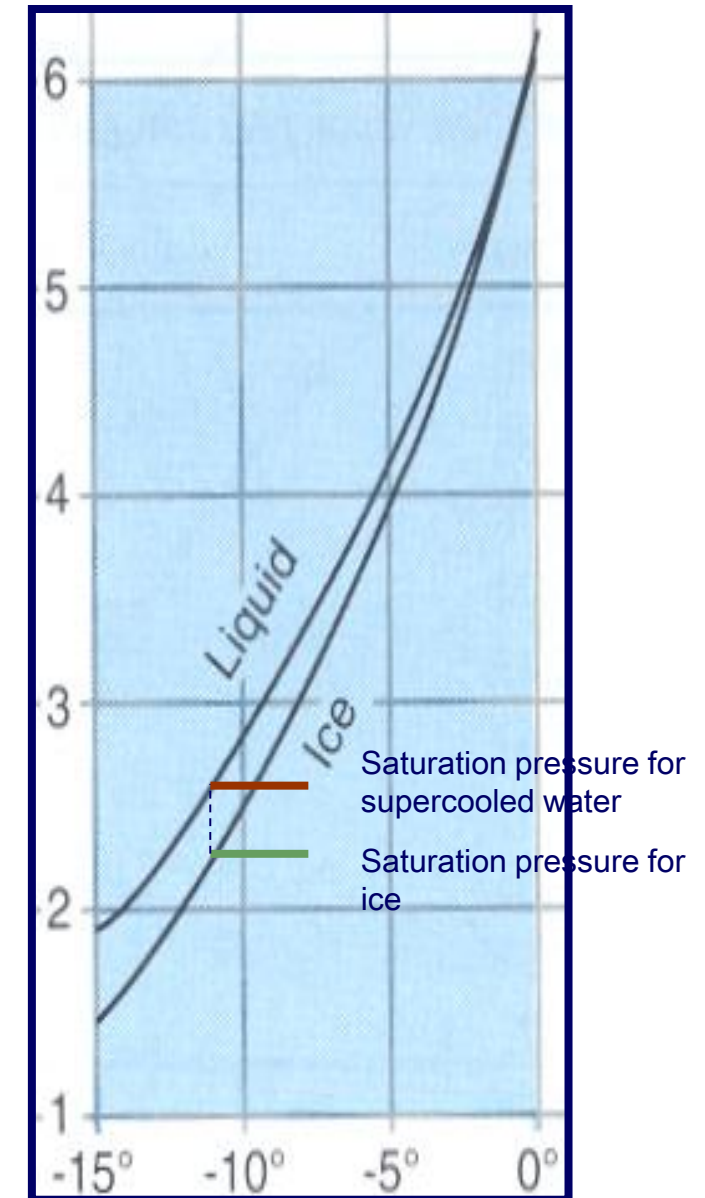
symbolic representation	EXAMPLES			NAME	SYMBOL
				plate	F1
				stellar crystal	F2
				column	F3
				needle	F4
				spatial dendrite	F5
				capped column	F6
				irregular crystal	F7
				graupel	F8
				ice pellet	F9
				hailstone	F0

Ice Crystal Growth in Clouds: nucleation

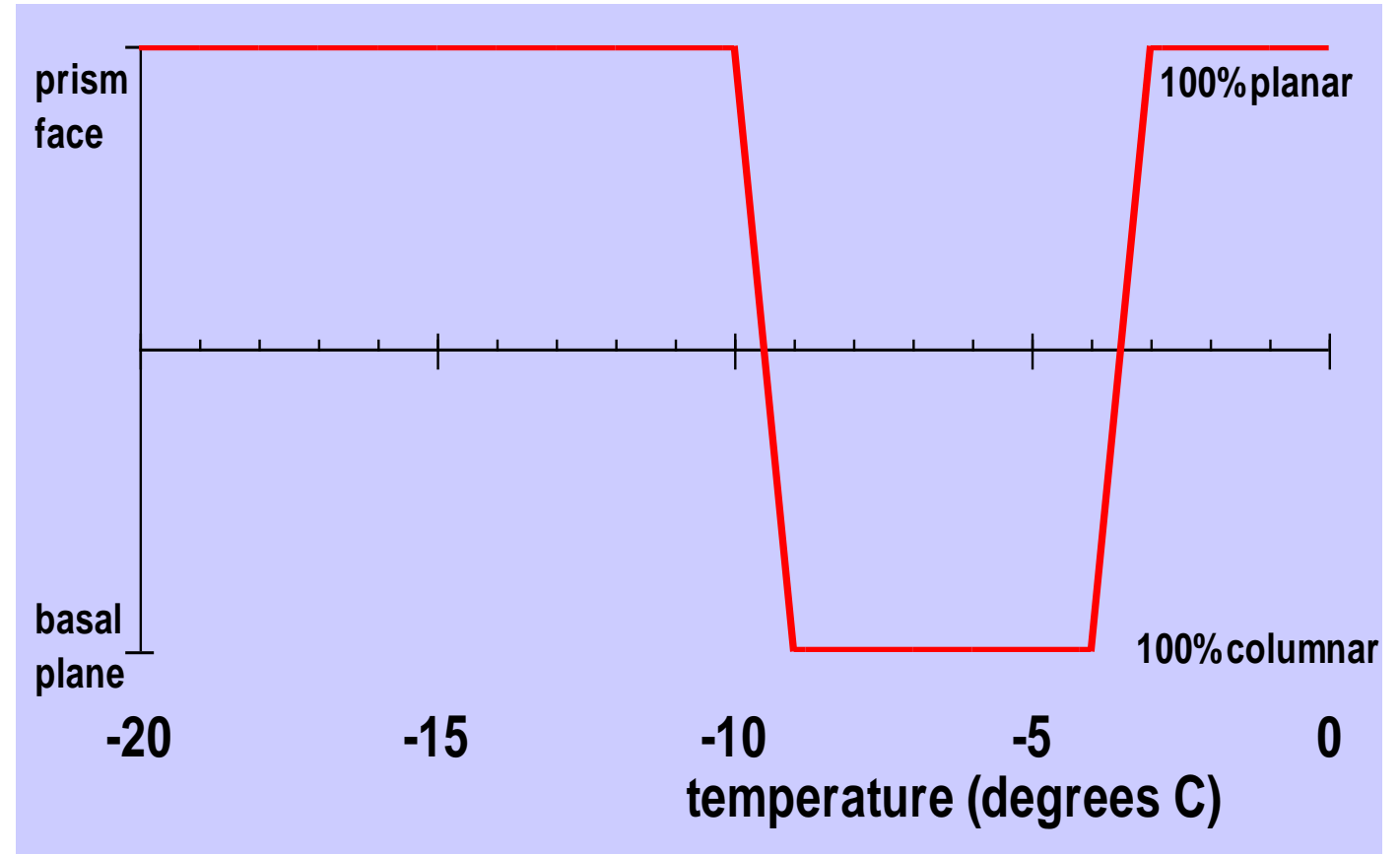
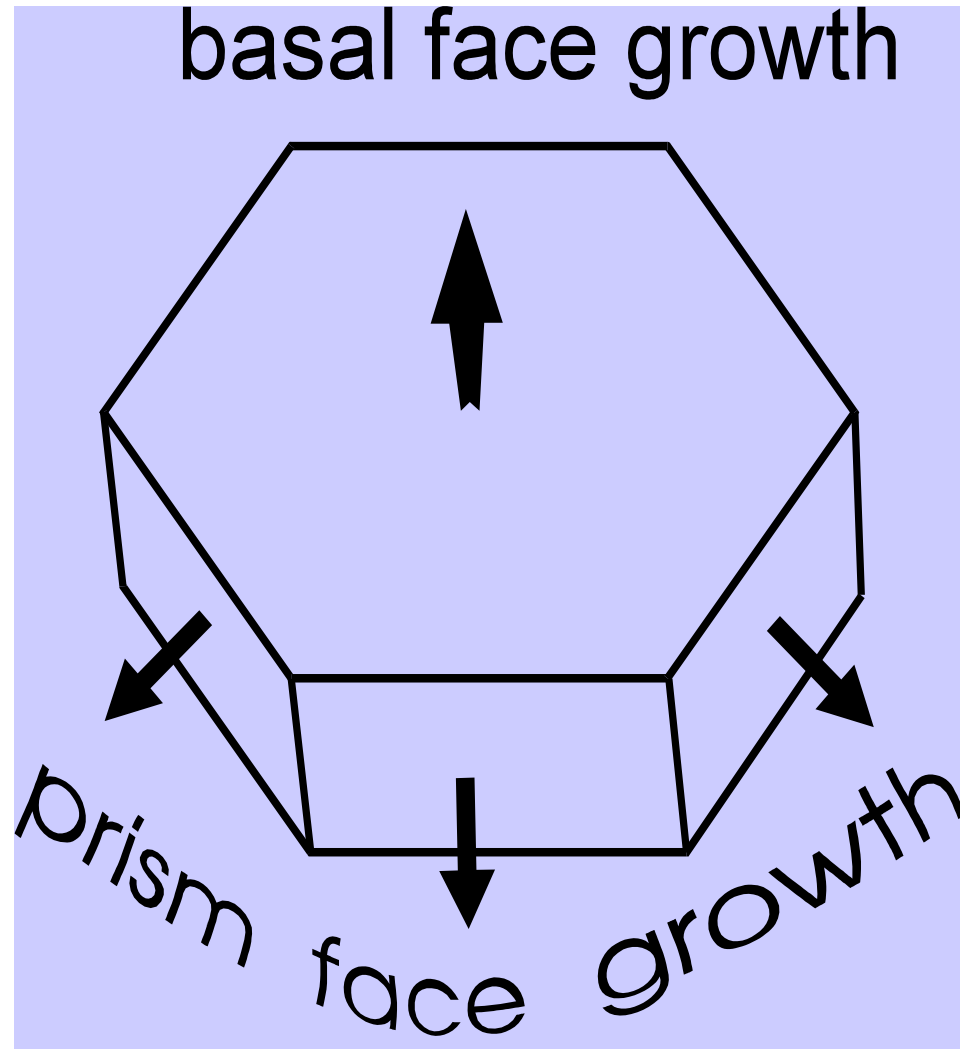
- spontaneous or homogeneous formation below $-40\text{ }^{\circ}\text{C}$
 - Inefficient as in the case of liquid cloud droplets
- heterogeneous nucleation
 - ice nucleus existing within a water droplet encourages freezing
 - water droplets aggregate around ice nucleus
 - temperature may be greater than $-40\text{ }^{\circ}\text{C}$
- contact nucleation
 - supercooled water comes in contact with ice nucleus and freezing occurs instantly
 - concentration of ice crystals within slightly supercooled convective clouds exceeds ice nuclei concentration by several orders of magnitude

Ice nucleation: mechanism of Bergeron-Findeisen

- This mechanism, which is completely due to the saturated vapour pressure differences between ice and supercooled water, requires the pre-existence of ice in the cloud
- There is divergence between saturation vapor pressure over ice and supercooled water at temperatures below 0 °C
 - Liquid vapor pressure is higher than ice vapor pressure at same temperature
- If air is supersaturated with ice, water vapour will deposit directly on existing ice particles
- air surrounding supercooled water droplets may become unsaturated with respect to liquid water (→ water droplets decrease) but remain saturated with respect to ice (→ ice particles grow)



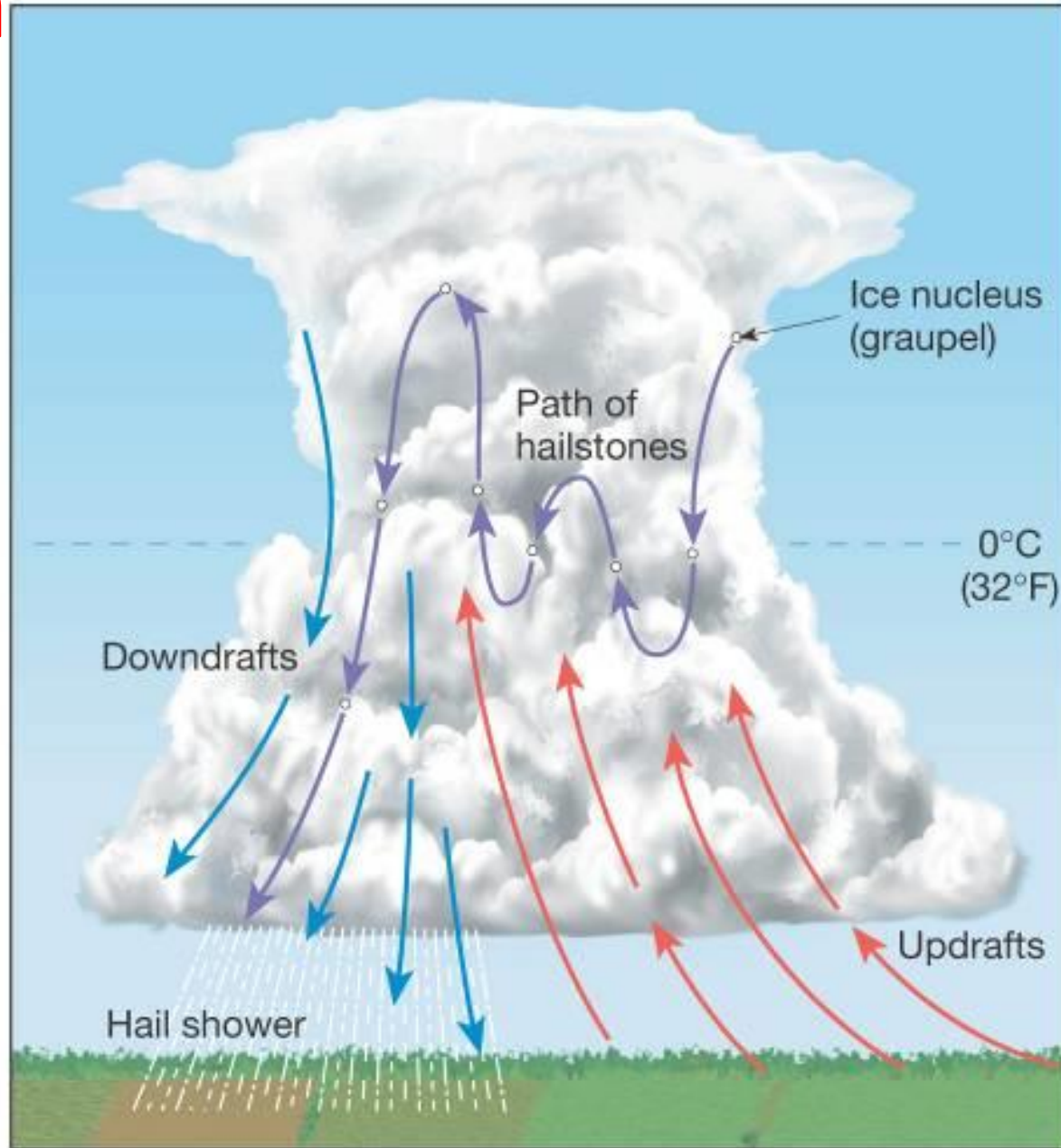
Snow Crystal shape is temperature dependent



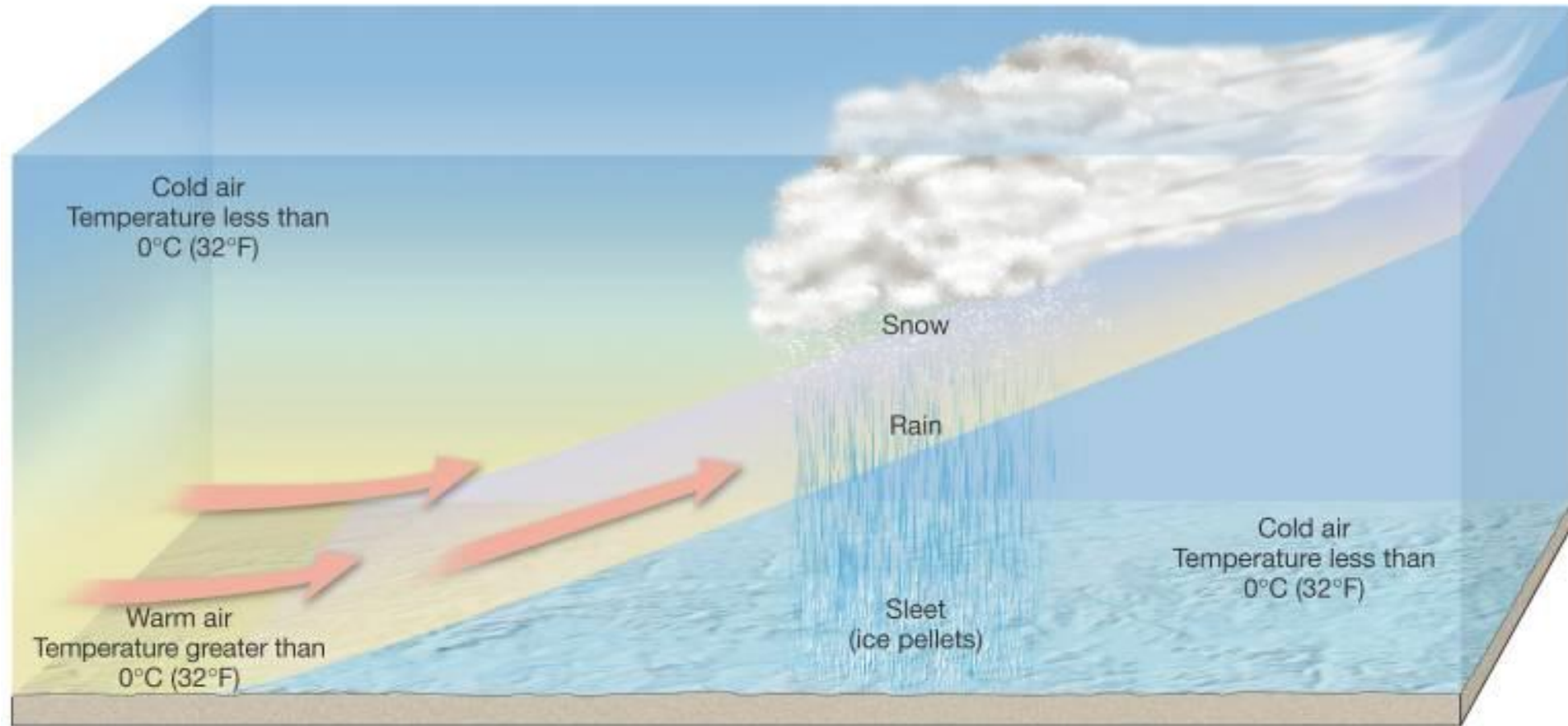
Growth for incorporation and freezing

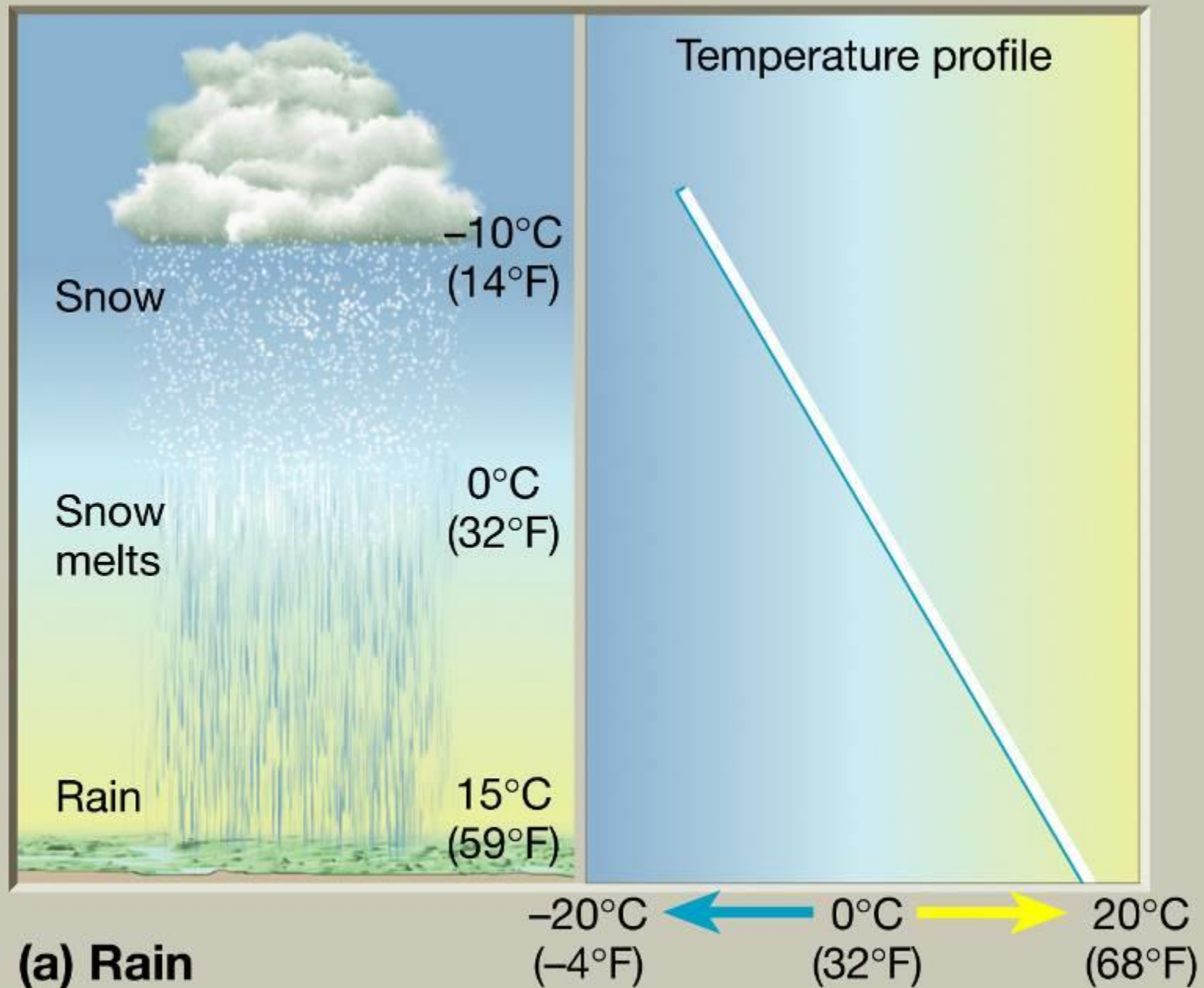
- Another efficient mechanism of growth is that **for incorporation and freezing** of supercooled water. This is usual for hailstones.
- There are 2 cases:
 - with light supercooling: water deposits on the ice crystal, then the water freezes → dense wet ice forms: riming, rimed snow crystals, graupel, soft hail, snow pellets
 - with high supercooling: there is instant freezing of vapor or supercooled water vapor on the ice crystal → dry fibrous ice forms: snow flakes, snow crystal, deposition
- The hailstones are often structured in alternating layers of fibrous and dense ice → index of strong vertical “oscillating” (up and down) movements
- Further mechanism of growth: **aggregation** of different ice crystals (typical of wet snowflakes) due to adhesive forces

Hail formation

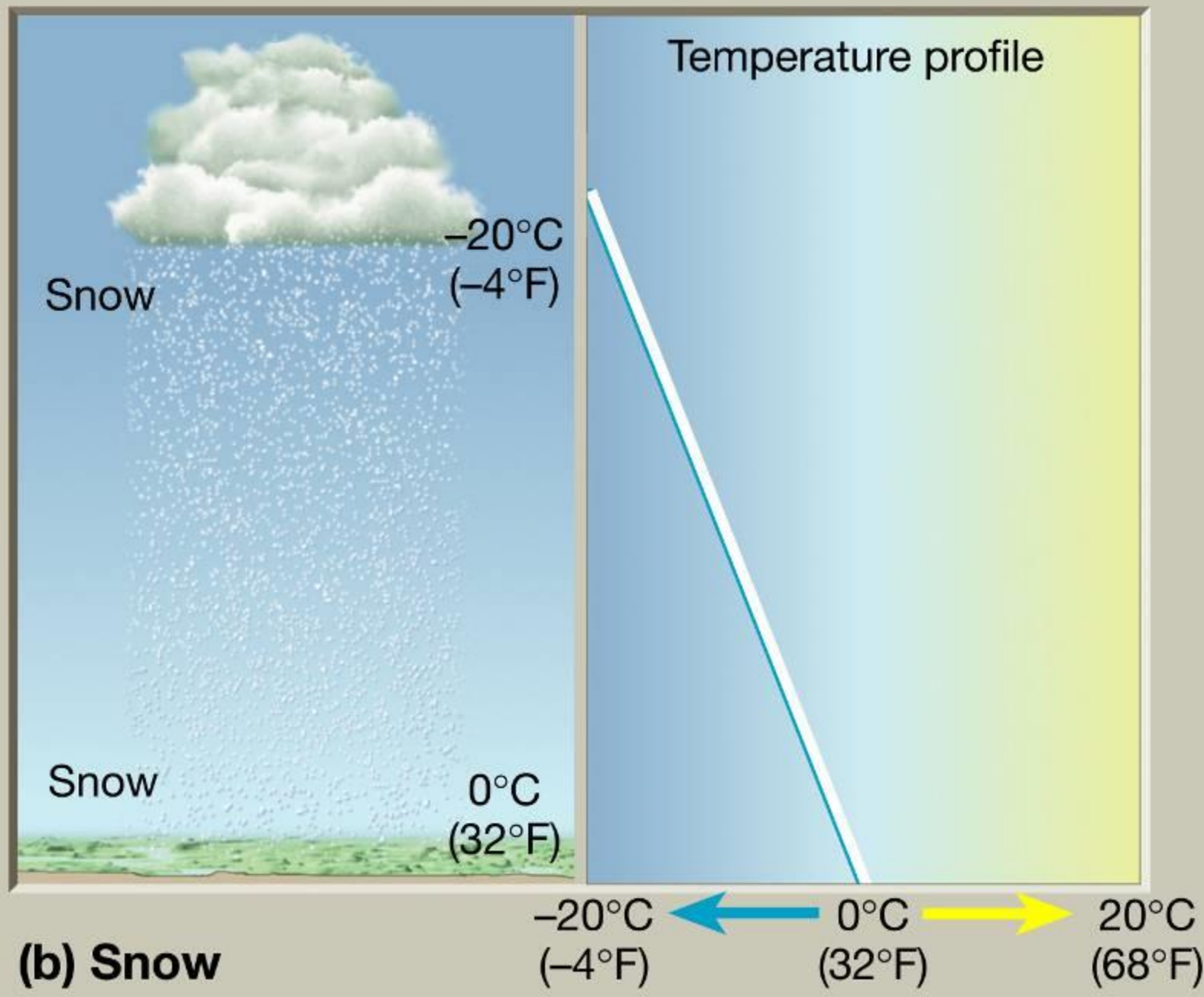


Conditions favoring sleet formation

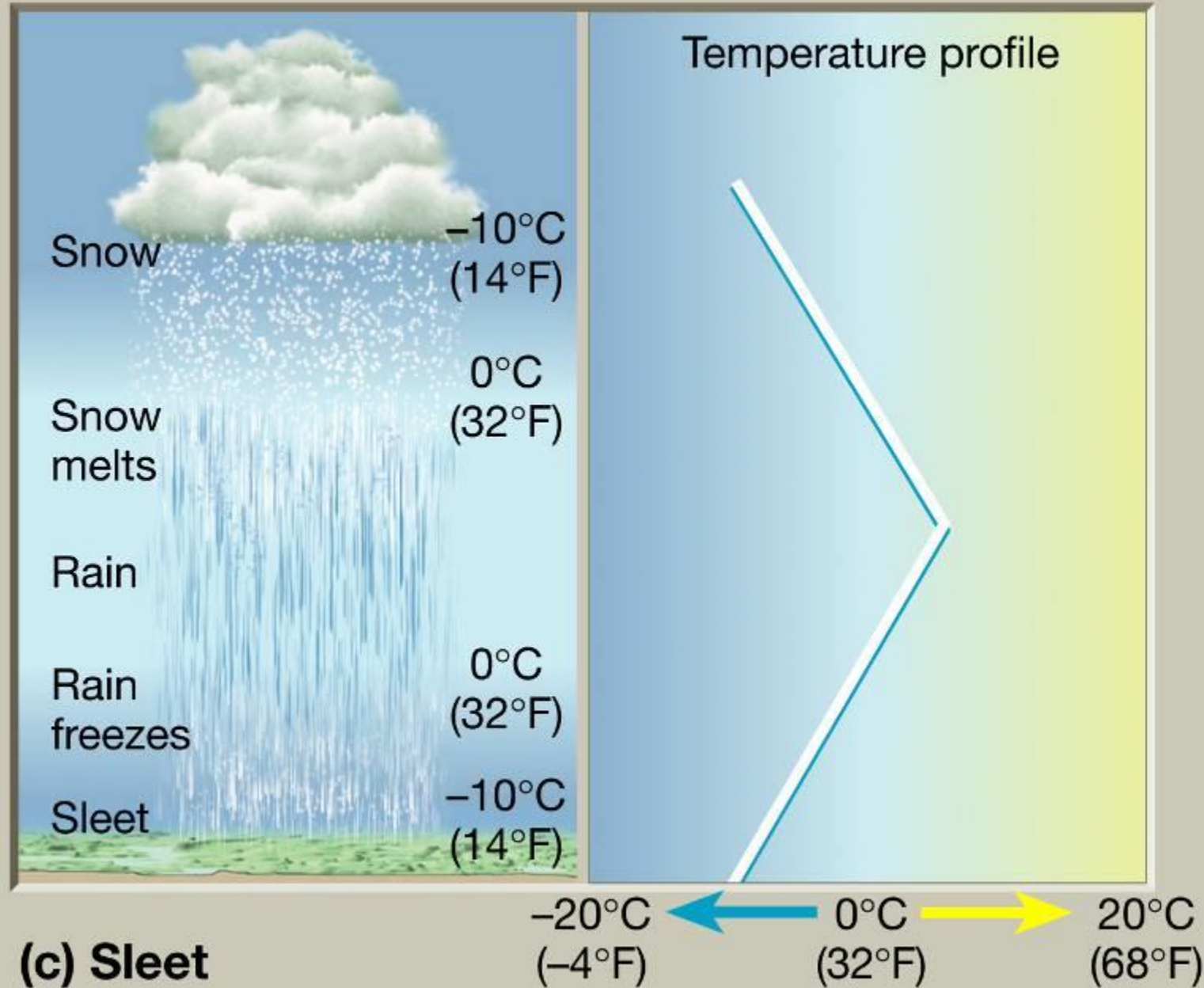


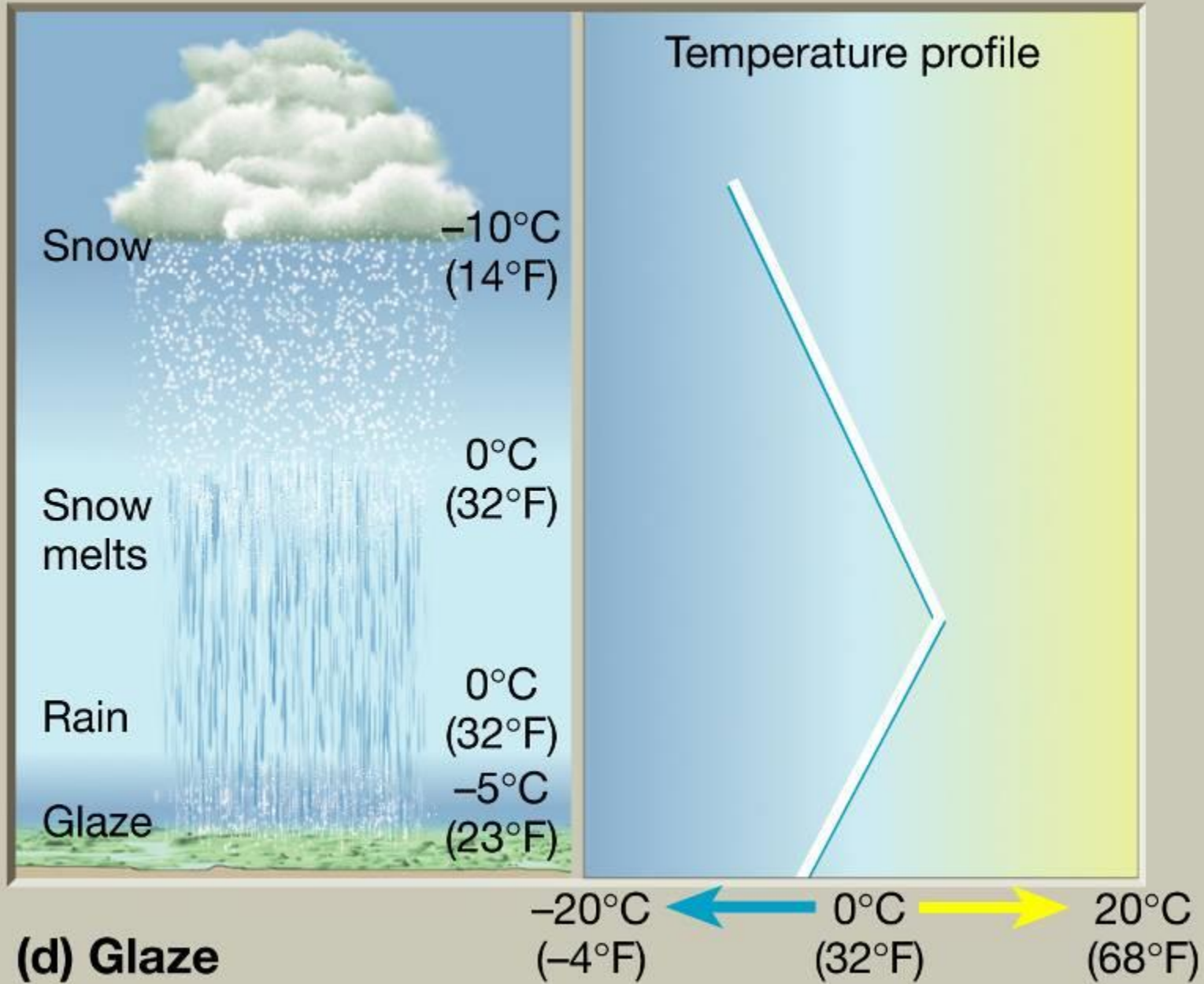


(a) Rain

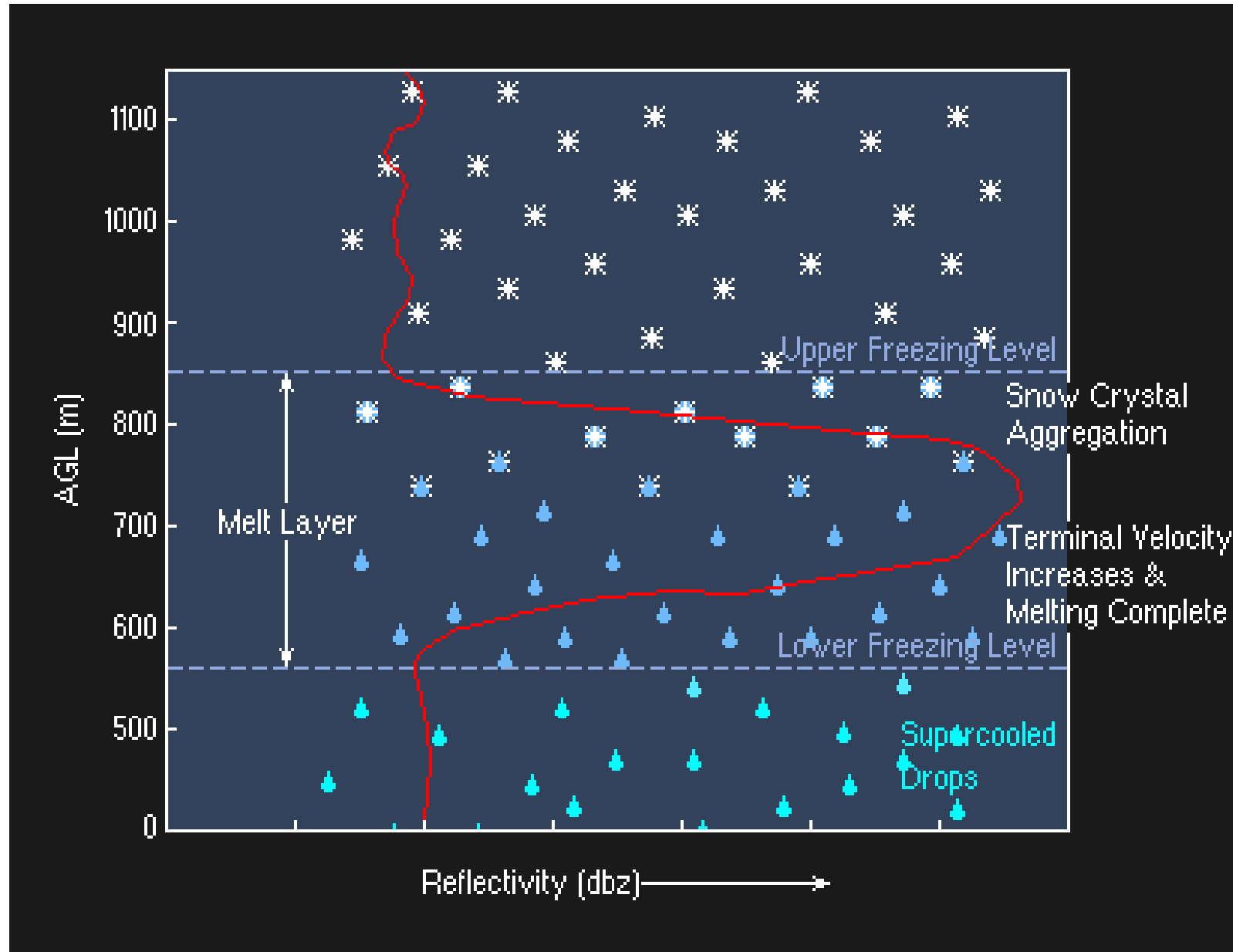


(b) Snow





The melting layer



Types of precipitation and characteristics

TABLE 5-4 Types of precipitation

Type	Approximate Size	State of Water	Description
Mist	0.005 to 0.05 mm	Liquid	Droplets large enough to be felt on the face when air is moving 1 meter/second. Associated with stratus clouds.
Drizzle	Less than 0.5 mm	Liquid	Small uniform drops that fall from stratus clouds, generally for several hours.
Rain	0.5 to 5 mm	Liquid	Generally produced by nimbostratus or cumulonimbus clouds. When heavy, size can be highly variable from one place to another.
Sleet	0.5 to 5 mm	Solid	Small, spherical to lumpy ice particles that form when raindrops freeze while falling through a layer of subfreezing air. Because the ice particles are small, any damage is generally minor. Sleet can make travel hazardous.
Glaze	Layers 1 mm to 2 cm thick	Solid	Produced when supercooled raindrops freeze on contact with solid objects. Glaze can form a thick coating of ice having sufficient weight to seriously damage trees and power lines.
Rime	Variable accumulations	Solid	Deposits usually consisting of ice feathers that point into the wind. These delicate frostlike accumulations form as supercooled cloud or fog droplets encounter objects and freeze on contact.
Snow	1 mm to 2 cm	Solid	The crystalline nature of snow allows it to assume many shapes, including six-sided crystals, plates, and needles. Produced in supercooled clouds where water vapor is deposited as ice crystals that remain frozen during their descent.
Hail	5 mm to 10 cm or larger	Solid	Precipitation in the form of hard, rounded pellets or irregular lumps of ice. Produced in large convective, cumulonimbus clouds, where frozen ice particles and supercooled water coexist.
Graupel	2 mm to 5 mm	Solid	Sometimes called “soft hail,” graupel forms as rime collects on snow crystals to produce irregular masses of “soft” ice. Because these particles are softer than hailstones, they normally flatten out upon impact.

Terminal Velocities

crystal type	dimension/diameter (mm)	terminal velocity (m/s)
Needle	1.53	0.50
Plane dendrite	3.26	0.31
Spatial dendrite	4.15	0.57
Powder snow	2.15	0.50
Crystals with graupel	2.45	1.00
Graupel	2.13	1.80
Rain drops	0.2	0.71
"	0.4	1.6
"	0.6	2.46
"	0.8	3.25
"	1.0	4.03
"	2.0	6.49
"	3.0	8.06
"	4.0	8.83
"	5.0	9.09

