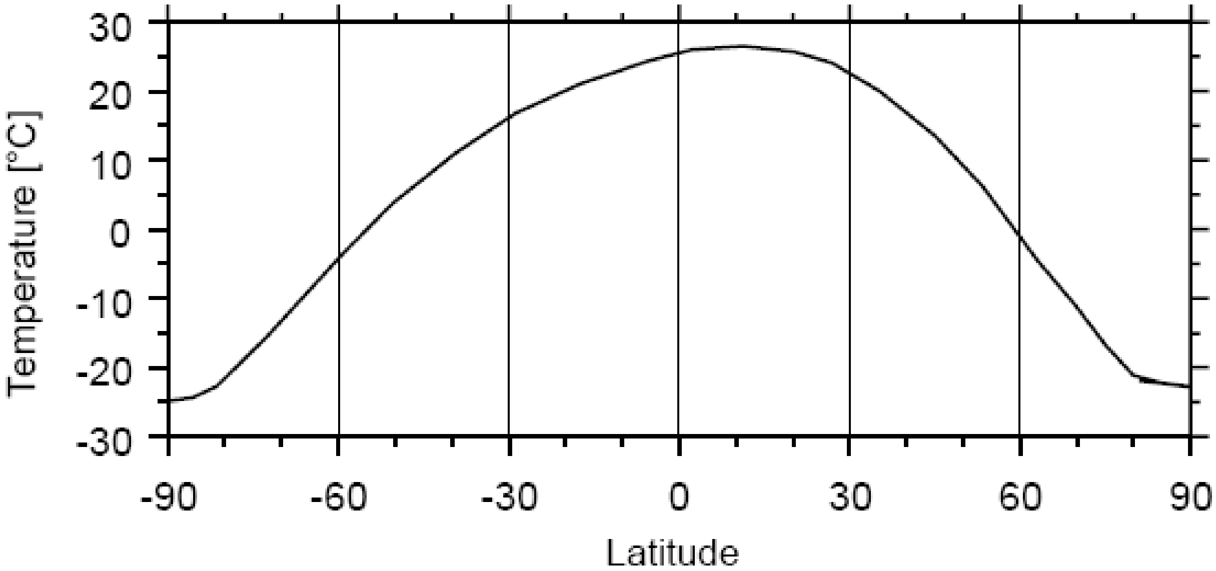
Thermal wind

Textbooks and web sites references for this lecture:

- A. Longhetto – Dispense di Fisica dell'atmosfera
- J. Holton – An introduction to dynamic meteorology
- Physic der atmosphäre - Institut für Umweltphysik - Universität Heidelberg
- Adrian M. Tomkins – Atmospheric Physics – ictp_atmospheric_physics.beamer.pdf

Thermal wind

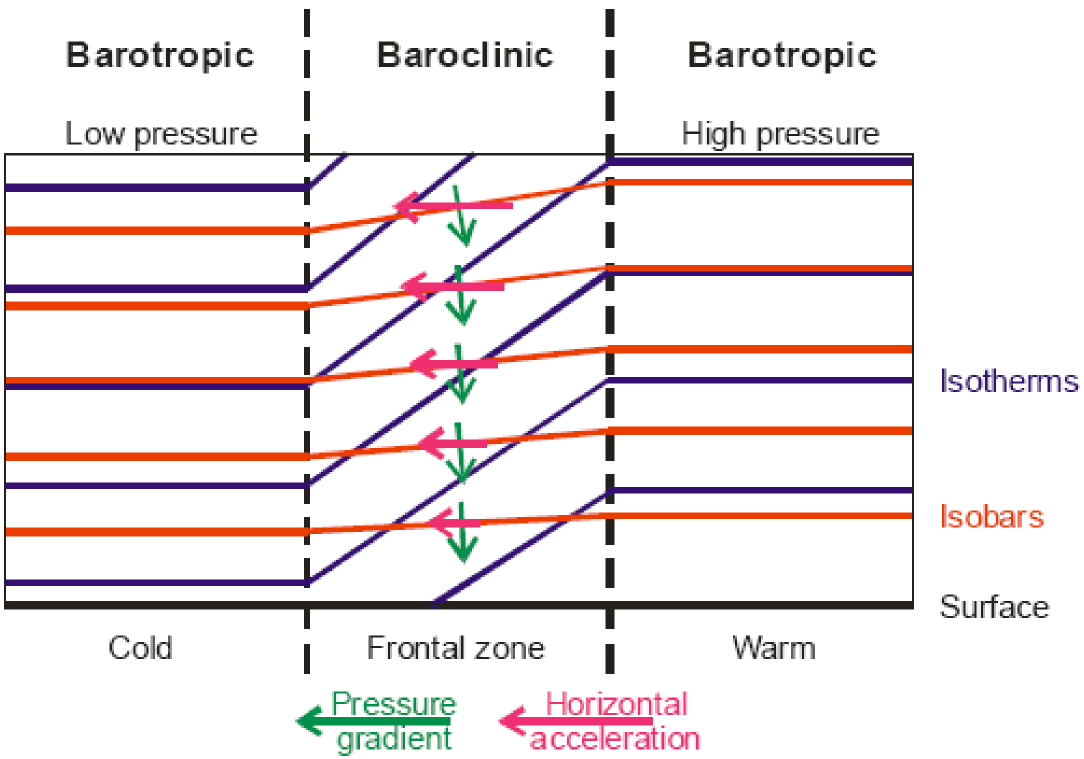
- Large-scale (e.g., from the Equator to the poles) and small scale (e.g, between land and water surfaces) temperature gradients have an impact on the atmospheric velocity field



- Which flow patterns occur due to temperature gradients?

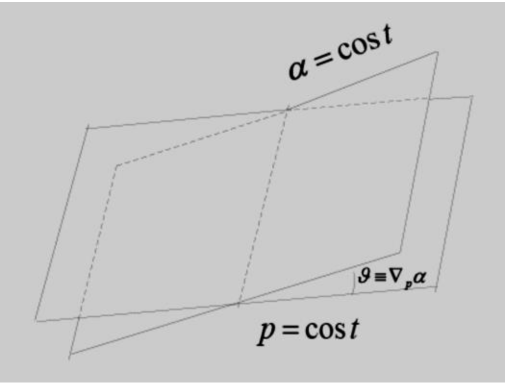
Barotropic and baroclinic atmosphere

- When $\nabla_p T \neq 0$ the atmosphere is said baroclinic; density is function of both p and T [$\rho=\rho(p,T)$], and isobaric surfaces does not coincide with iso-density surfaces
- When $\nabla_p T = 0$ the atmosphere is said barotropic



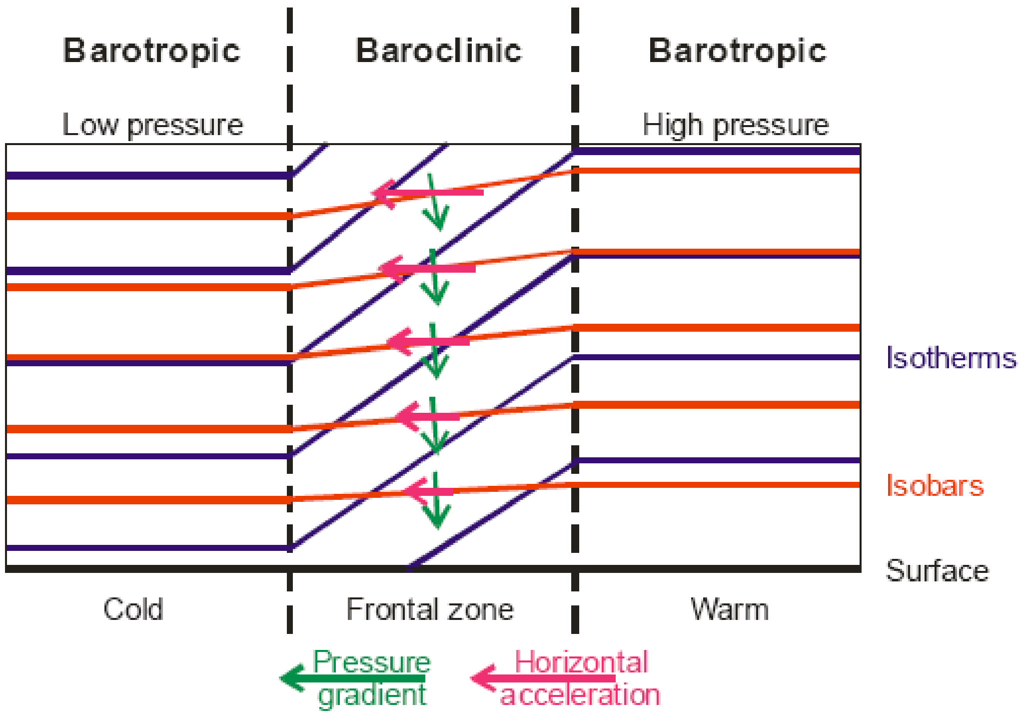
$$\nabla_p \alpha = \nabla_p \left(\frac{1}{\rho}\right) = \nabla_p \left(\frac{R_d T}{p}\right)$$

$$\xrightarrow{p=const} \frac{R_d}{p} \nabla_p T$$



Baroclinic atmosphere

- Assume two areas with different surface temperature, separated by a frontal zone
- Isotherms and isobars are parallel over cold and warm areas. This situation is called barotropic
- Pressure over cold area decreases faster with height, due to hydrostatic equation: $dp = -\rho g\, dz = -(pg/R_d T)\, dz$
- Thus isobars are slanted in frontal zone, leading to a horizontal component of the pressure gradient and thus a horizontal acceleration

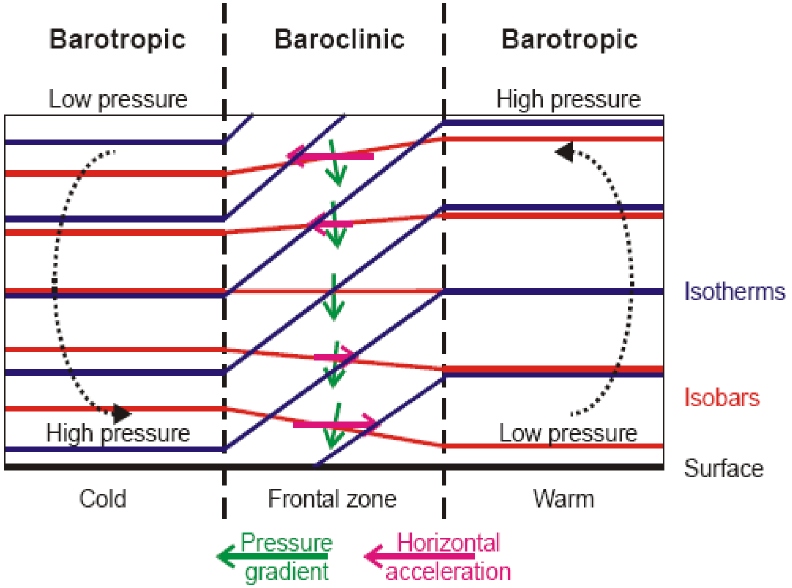


- Isotherms are slanted even stronger
- →Isobars and isotherms are not parallel in frontal zone (baroclinic)
- Baroclinic stratification leads to acceleration of air

Thermal wind - direct thermal circulation

- Temperature differences on a small scale (e.g., sea/land surface)
- Coriolis force can be neglected
- Air flow at high altitudes from warm to cold area due to pressure difference
- This induces:
  - upward movement of air over the warm area
  - downward movement of air over the cold area

- Thus higher surface pressure over the cold area than over the warm area
- Therefore surface wind blows from cold to warm area
- This circulation pattern is called direct thermal circulation



Thermal wind - direct thermal circulation

- Quantitative description of thermal wind: Consider the circulation of the wind vector field:

$$Z = \oint \vec{v} \cdot d\vec{s}$$

- Note: the circulation Z is connected to the rotation of a vector field via

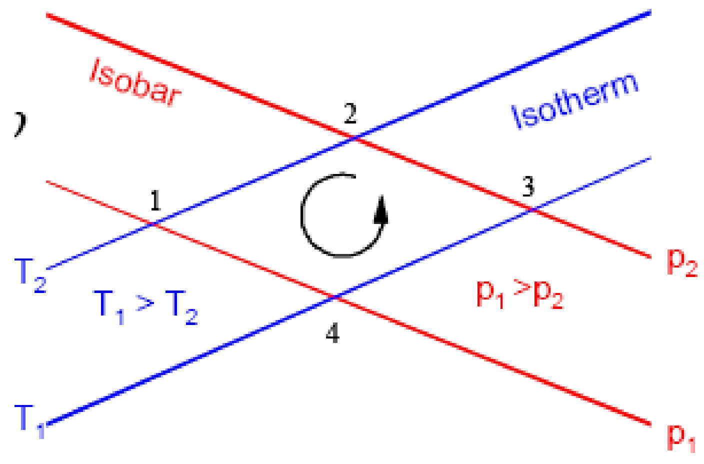
$$\nabla \times \vec{v} = \lim_{A \to 0} \frac{1}{A} \oint \vec{v} \cdot d\vec{s}$$

- Acceleration by pressure gradients:

$$\frac{dZ}{dt} = \frac{d}{dt} \oint \vec{v} \cdot d\vec{s} = \oint \vec{a} \cdot d\vec{s} = -\frac{1}{\rho} \oint \overrightarrow{\nabla p} \cdot d\vec{s} = -\oint \frac{dp}{\rho}$$

- Separation of the path integral into the sections shown in the figure:

$$\frac{dZ}{dt} = -\left(\int_1^2 \frac{dp}{\rho} + \int_2^3 \frac{dp}{\rho} + \int_3^4 \frac{dp}{\rho} + \int_4^1 \frac{dp}{\rho} \right)$$



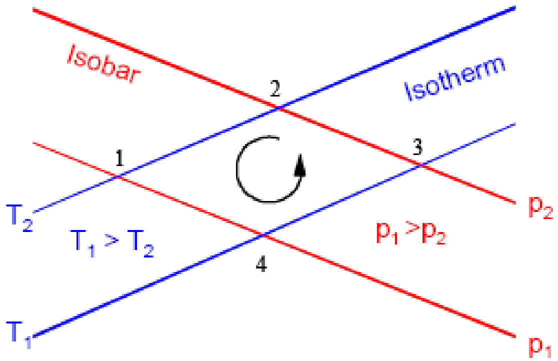
Thermal wind - direct thermal circulation

- The second and forth integrals are zero since dp= 0 on isobars. With the ideal gas law, ρ= p/(RT), this yields:

$$\frac{dZ}{dt} = -R\left(T_2 \int_{p_1}^{p_2} \frac{dp}{p} + T_1 \int_{p_2}^{p_1} \frac{dp}{p} \right)$$

- Integration yields:

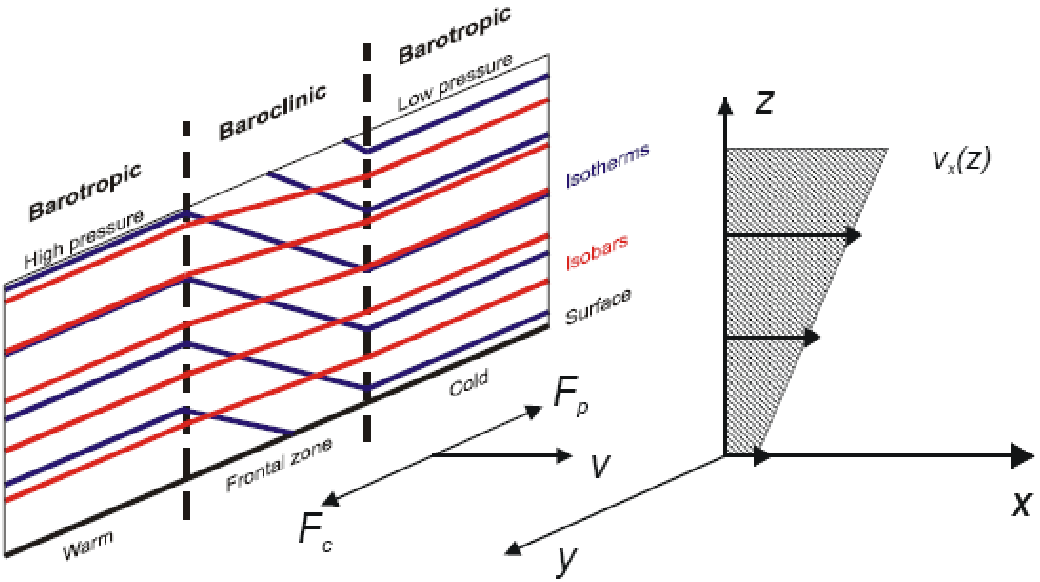
$$\frac{dZ}{dt} = R(T_2 - T_1)\ln \frac{p_1}{p_2}$$



- →Baroclinic stratification leads to a circulation of the wind vector field
- In the example shown in the figure, a counterclockwise rotation of the velocity field occurs

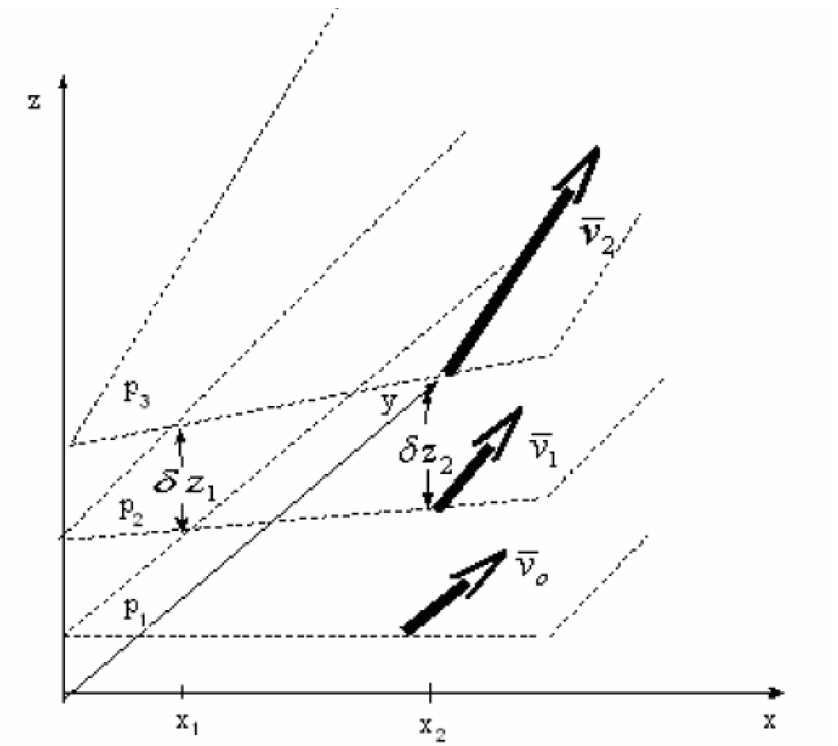
Thermal wind and geostrophic wind

- Temperature differences on a large scale (e.g., latitudinal temperature gradients)
- Coriolis force cannot be neglected
- This leads to a geostrophic air flow
- Increase in altitude (geopotential height) difference with decreasing pressure leads to an increase in wind speed with height



Differential equation of thermal wind

- Let's consider a geostrophic wind changing both module and direction with height



$$\overrightarrow{u_g} = \frac{\vec{k} \times \overrightarrow{\nabla_p \Phi}}{f}$$

$$\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho} = -\frac{R_d T}{p}$$

$$\frac{\partial \overrightarrow{u_g}}{\partial p} = \frac{\vec{k}}{f} \times \frac{\partial \overrightarrow{\nabla_p \Phi}}{\partial p} = \frac{\vec{k}}{f} \times \overrightarrow{\nabla_p \left(\frac{\partial \Phi}{\partial p} \right)} =$$

$$= -\frac{R_d}{fp} \vec{k} \times \overrightarrow{\nabla_p T}$$

$$\boxed{\frac{\partial \overrightarrow{u_g}}{\partial \ln p} = -\frac{R_d}{f} \vec{k} \times \overrightarrow{\nabla_p T}}$$

- This is known as the **differential equation of thermal wind** and express the vertical wind shear with respect to the geostrophic wind defined at a certain reference level

The thermal wind

- Thermal wind is defined as the difference between geostrophic wind at two different levels
- In barotropic atmosphere, thermal wind is null, as geostrophic wind is constant with the heigth
- In baroclinic atmosphere, integrating previous equation between two levels p_0 and p_1, we get

$$\overrightarrow{v_T} = \int_0^1 du_g = \overrightarrow{u_{g1}} - \overrightarrow{u_{g0}} = \frac{R_d}{f}\vec{k} \times \overrightarrow{\nabla_p \overline{T}} \ln \frac{p_0}{p_1}$$

- From the definition of thermal wind:

$$\overrightarrow{v_T} = \overrightarrow{u_g(p_1)} - \overrightarrow{u_g(p_0)} = \frac{\vec{k}}{f} \times \overrightarrow{\nabla_p(\Phi_1 - \Phi_0)}$$

- These two equations are equivalent:

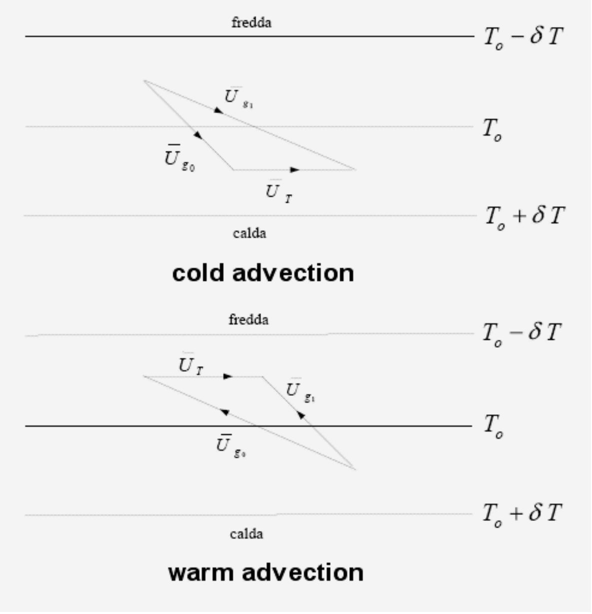
$$\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho} = -\frac{R_d T}{p} \quad \Rightarrow \quad d\Phi = -R_d T d\ln p \quad \Rightarrow$$

$$\Rightarrow \quad \Phi_1 - \Phi_0 = -R_d \overline{T} \ln\left(\frac{p_0}{p_1}\right) \quad \Rightarrow \quad \nabla(\Phi_1 - \Phi_0) = -R_d \nabla\overline{T} \ln\left(\frac{p_0}{p_1}\right)$$

The thermal wind

- The quantity $\Delta\Phi=\Phi_1-\Phi_0$ is called **depth of the p_0-p_1 layer**, and is proportional to the mean temperature of the layer
  - The lines Φ=const represent mean isotherms
- The quantity Φ/gz represents approximately the depth in heigth of geopotential
- The vectorial form of thermal wind $\vec{v_T} = \frac{\vec{k}}{f} \times \overrightarrow{\nabla_p(\Phi_1 - \Phi_0)}$

  shows that its direction is parallel to the isotherms, with warm air on the right



- If geostrophic wind varies with heigth anticlockwise, it is possible to evaluate thermal wind vector parallel to the isotherms and see that geostrophic wind pushes cold air (cold advection) – and viceversa
- Thus, just observing how geostrophic wind rotates with the heigth, it is possible to deduce the temperature field and the type of advection

Summary on thermal wind

- Thermal wind occurs due to horizontal temperature gradients
  - Direct thermal wind is a small scale process
    - Coriolis force can be neglected
    - It leads to a circulation, with wind blowing from cold to warm surfaces
  - Geostrophic thermal wind occurs on large scales (e.g., latitudinal temperature gradient)
    - It causes a geostrophic wind pattern
    - The wind speed increases linearly with altitude